Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2022/2023

COURSE NAME	:	ENGINEERING MATHEMATICS
COURSE CODE	:	DAM13303
PROGRAMME CODE	:	DAM
EXAMINATION DATE	:	JULY / AUGUST 2023
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWER ALL QUESTIONS. 2. THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK**. 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK.

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

Q1 (a) Sketch and identify the domain and range of function $p(x) = -x^2 + 10x - 3$.

(4 marks)

(b) The function $k(r)$ is given by below:

$$k(r) = \begin{cases} -r^2 - 4r & , \quad -5 \le r < -2 \\ r + 6 & , \quad -2 \le r \le 0 \\ -2r^3 + 6 & , \quad 0 \le r \le 2 \\ -10 & , \quad r \ge 2 \end{cases}.$$

(i) Sketch the graph of $k(r)$.

(5 marks)

(ii) State the domain and range of function $k(r)$.

(2 marks)

(iii) Justify the piecewise function above if there is any continuity at $r = 0$.

(3 marks)

(c) Functions are given by $f(x) = 2x + 1$, $g(x) = x^2 - 3$ and $h(x) = \dfrac{mx}{x+1}$. Find the value of m if $\left[h \circ f \circ g\right](2) = \dfrac{3}{2}$.

(4 marks)

(d) Evaluate the following limit:

(i) $\displaystyle\lim_{x \to 5} \frac{\sqrt{x+4} - 2}{x}$.

(4 marks)

(ii) $\displaystyle\lim_{x \to -4} \frac{x^2 + x - 12}{x^2 + 5x + 4}$.

(3 marks)

Q2 (a) Find the derivatives of $\left(\dfrac{5t^2}{3t^2 + 2}\right)^3 + \ln t$. Give your answer in the simplest form.

(5 marks)

(b) Differentiate the following function using implicit differentiation method:

$$2x^{6/3} + 8y^2 - 4xy = 12.$$

(5 marks)

(c) Given parametric functions, $x = \frac{t+2}{t}$ and $y = \frac{t-2}{t}$. Find $\frac{dy}{dx}$.

(4 marks)

(d) Zamir dips the end of his paintbrush, and a circular patch of colour forms around the glass. Calculate the rate of change of the area, $A = \pi r^2$ if the radius of the paint is growing at a constant rate of 30mm/second. Then compute the patch's area when its radius is 100mm.

(4 marks)

(e) Determine the absolute extreme values of the following function on the given interval:

$$f(x) = \sin 3x \text{, where } -45° \leq x \leq 60°.$$

(7 marks)

Q3 (a) By using integration by part, solve $\int e^{2t} \cos\left(\frac{1}{4}t\right) \, dt$.

(7 marks)

(b) Solve $\int_{-1}^{3} x(x^2 + \sqrt{5})^3 \, dx$ by using substitution method.

(8 marks)

(c) The region ***R*** bounded by $y = x^2 - 7x + 9$ and $y = x + 4$.

(i) Sketch the region ***R***.

(3 marks)

(ii) Find the area of the region ***R*** bounded by the two curves.

(7 marks)

Q4 (a) Solve the following separable differential equation:

$$y' = \frac{x^2 y^4}{1 + x^3}.$$

(7 marks)

(b) Given a differential equation:

$$\left(2x^2 + 15xy + 5y^2\right)dx - (4xy + 10x^2)dy = 0.$$

(i) Show that the differential equation above is a homogeneous equation.

(3 marks)

(ii) Then, solve the given differential equation.

(7 marks)

(c) A culture initially has N_0 number of bacteria. At time $t = 1$ hour the number of bacteria is measured to be $\frac{3}{2}N_0$. If the rate of growth is proportional to the number of bacteria present at any time, determine the time necessary for the number of bacteria to triple.

(8 marks)

- END OF QUESTIONS -

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Formula

Table 1: Integration and Differentiation

Integration	Differentiation
$\int x^n dx = \dfrac{x^{n+1}}{n+1} + C$	$\dfrac{d}{dx} x^n = nx^{n-1}$
$\int \dfrac{1}{x} dx = \ln\lvert x\rvert + C$	$\dfrac{d}{dx} \ln x = \dfrac{1}{x}$
$\int \dfrac{1}{a-bx} dx = -\dfrac{1}{b} \ln\lvert a-bx\rvert + C$	$\dfrac{d}{dx} \ln(u) = \dfrac{1}{u}\dfrac{du}{dx}$ **where** $u = f(x)$
$\int e^{ax+b} dx = \dfrac{1}{a} e^{ax+b} + C$	$\dfrac{d}{dx} e^u = e^u \dfrac{du}{dx}$
$\int \sin(ax+b)\, dx = -\dfrac{1}{a}\cos(ax+b) + C$	$\dfrac{d}{dx} \sin u = \cos u \dfrac{du}{dx}$
$\int \cos(ax+b) dx = \dfrac{1}{a}\sin(ax+b) + C$	$\dfrac{d}{dx} \cos u = -\sin u \dfrac{du}{dx}$
$\int \sec^2(ax+b)\, dx = \tan(ax+b) + C$	$\dfrac{d}{dx} \tan u = \sec^2 u \dfrac{du}{dx}$
$\int \csc^2(ax+b)\, dx = -\cot(ax+b) + C$	$\dfrac{d}{dx} \cot u = -\csc^2 u \dfrac{du}{dx}$
$\int u\, dv = uv - \int v du$	$\dfrac{d}{ds}(uv) = u\dfrac{dv}{ds} + v\dfrac{du}{ds}$
$\int_a^b f(x) dx = F(b) - F(a)$	$\dfrac{d}{ds}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{ds} - u\dfrac{dv}{ds}}{v^2}$
	Chain Rule: $\dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx}$
	Parametric Differentiation: $\dfrac{dy}{dx} = \dfrac{dy}{dt} \cdot \dfrac{dt}{dx}$

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FORMULA

Table 2: Area and Volume

Area of Region
$A=\int_a^b [f(x)-g(x)]\,dx$ or $A=\int_c^d [f(y)-g(y)]\,dy$
Volume Cylindrical Shells
$V=\int_a^b 2\pi x[f(x)-g(x)]\,dx$ or $V=\int_c^d 2\pi y[f(y)-g(y)]\,dy$