UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2014/2015

COURSE NAME	:	ENGINEERING MATHEMATICS II
COURSE CODE	:	DAS 20403
PROGRAMME	:	2 DAA / 2 DAM / 3 DAA / 3 DAM
EXAMINATION DATE	:	DECEMBER 2014/JANUARY 2015
DURATION	:	3 HOURS
INSTRUCTION	:	A) ANSWER **ALL** QUESTIONS IN **SECTION A**. B) ANSWER **THREE (3)** QUESTIONS ONLY IN **SECTION B** .

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

SECTION A

Q1 (a) Find the inverse Laplace of following transform.

(i) $\frac{7}{s^3}-\frac{5}{s}+\frac{1}{s+3}$

(6 marks)

(ii) $\frac{4s}{4s^2+25}$

(5 marks)

(b) (i) Express $\frac{3s}{s^2+s-12}$ as partial fraction.

(6 marks)

(ii) Find the inverse Laplace of the partial fraction from **Q1(b)(i)**.

(3 marks)

Q2 Solve the given initial and boundary value problem of differential equation using Laplace transform.

(a) $y''-3y'-10y=0$; Initial value problem : $y(0)=0,\ y'(0)=-5$

(8 marks)

(b) $y''-7y'+12y=2$; Boundry value problem : $y=1,\ y'=5,$ when $t=0$.

(12 marks)

SECTION B

Q3 (a) Given

$$(x+y)^2\,dx+\left(2xy+x^2-1\right)dy=0$$

(i) Show that the equation is an exact ordinary differential equation.

(3 marks)

(ii) Find the general solution of the equation.

(8 marks)

(b) Find the solution of the given IVP differential equation.

$$x\frac{dy}{dx}+2y=\cos x \quad ; \quad y(1)=2$$

(9 marks)

Q4 (a) The temperature change of an object according to Newton's Law of cooling is given as follows:

$$\frac{dT}{dt} = -k(T_t - T_s)$$

where T_t is the object temperature at time t, T_s is the surrounding temperature, and k is the cooling constant. By integration of the separable differential equation, express T_t in term of T_s, t and k.

(5 marks)

(b) Water is heated to a boiling point temperature of 100 °C. It is then removed from the burner and kept in a room which the surrounding temperature of 25 °C. After 10 minutes, the temperature of the hot water drops to 90 °C. Assuming that there is no change in the room temperature and the cooling processes followed the Newton's Law of cooling;

(i) List temperature of the surrounding (T_s), temperature at $t = 0$ minute (T_0), and temperature at $t = 10$ minutes (T_{10}).

(3 marks)

(ii) Using the formula of T_t from **Q4(a)**, calculate the time needed for the hot water to reach 30 °C?

(12 marks)

Q5 (a) Using undetermined coefficient method, solve the equation

$$y'' - 3y' + 2y = 6e^{5x},$$

given that when $y(0) = 0$ and $y'(0) = 1$.

(10 marks)

(b) Using variations of parameter method, solve the equation

$$y'' - y' - 2y = e^{2x}.$$

(10 marks)

Q6 (a) Find the Laplace Transforms of the following functions.

(i) $f(t)=\left(\cos 2t+\frac{1}{4}\sin 2t\right)e^{2t}$

(5 marks)

(ii) $f(t)=e^{2t}(t-2)(t-3)$

(5 marks)

(iii) $f(t)=(1-2\cos t)^2$

(5 marks)

(b) Show that $\mathcal{L}\left\{t^2\sin 2t\right\}=\frac{4(3s^2-4)}{(s^2+4)^3}$.

(5 marks)

Q7 (a) Solve the second order homogeneous differential equation.

$$y''-2y'-3y=0 \quad ; \quad y(0)=2 \quad \text{and} \quad y'(0)=1$$

(7 marks)

(b) Find the Inverse Laplace Transform of $\frac{1}{s+5}-\frac{12}{s^3}+\frac{1}{s^5}$.

(5 marks)

(c) Use Laplace Transform to solve the differential equation $y'+y=\cos t$ given $y(0)=0$.

(8 marks)

– END OF QUESTION –

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Formula

Table 1 : Laplace Transform.

$\mathcal{L}\{f(t)\}=\int_0^\infty f(t)e^{-st}\,dt = F(s)$	
$f(t)$	$F(s)$
k	$\frac{k}{s}$
t^n, $n = 1, 2, ..$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
$e^{at} f(t)$	$F(s-a)$
$t^n f(t)$, $n = 1, 2, ..$	$(-1)^n \frac{d^n F(s)}{ds^n}$

Table 2 : Indefinite Integration/Differentiation

Integration
$\int x^n dx = \frac{x^{n+1}}{n+1} + C$
$\int \frac{1}{x} dx = \ln\lvert x\rvert + C$
$\int \frac{1}{a-bx} dx = -\frac{1}{b}\ln\lvert a-bx\rvert + C$
$\int e^{ax} dx = \frac{1}{a}e^{ax} + C$
$\int \sin ax\, dx = -\frac{1}{a}\cos ax + C$
$\int \cos ax\, dx = \frac{1}{a}\sin ax + C$
$\int u dv\, dx = uv - \int v du$
Differentiation
$\frac{d}{ds}(uv) = v\frac{du}{ds} + u\frac{dv}{ds}$
$\frac{d}{ds}\left(\frac{u}{v}\right) = \frac{v\frac{du}{ds} - u\frac{dv}{ds}}{v^2}$
$\frac{d}{ds}\left(e^{ax}\right) = ae^{ax}$
$\frac{d}{ds}(\sin ax) = a\cos ax$
$\frac{d}{ds}(\cos ax) = -a\sin ax$
$\frac{d}{ds}\left(x^n\right) = nx^{n-1}$

Table 3 : Initial and Boundary Value Problem

If $L\{y(t)\} = Y(s)$ then

$L\{y'(t)\} = sY(s) - y(0)$

$L\{y''(t)\} = s^2Y(s) - sy(0) - y'(0)$

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Table 4 : Characteristic Equation and General Solution

Differential equation : $ay'' + by' + cy = 0$;
Characteristic equation : $am^2 + bm + c = 0$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Case	Roots of the Characteristic Equation	General Solution
1	real and distinct : $m_1 \neq m_2$	$y_h(x) = Ae^{m_1 x} + Be^{m_2 x}$
2	real and equal : $m_1 = m_2 = m$	$y_h(x) = (A + Bx)e^{mx}$
3	imaginary : $m = \alpha \pm i\beta$	$y_h(x) = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Table 5 : Solution of particular solution $ay'' + by' + cy = f(x)$

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + A_{n-1} x^{n-1} + \,..+ A_1 x + A_0$	$x^r (B_n x^n + B_{n-1} x^{n-1} + ..+ B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r (Pe^{\alpha x})$
$C\cos\beta x$ atau $C\sin\beta x$	$x^r (p \text{ kos } \beta x + q \sin \beta x)$
$P_n(x)e^{\alpha x}$	$x^r (B_n x^n + B_{n-1} x^{n-1} + ..+ B_1 x + B_0)\, e^{\alpha x}$
$P_n(x)\begin{cases} kos\beta x & atau \\ \sin\beta x \end{cases}$	$x^r (B_n x^n + B_{n-1} x^{n-1} + ..+ B_1 x + B_0)\, kos\,\beta x$ + $x^r (B_n x^n + B_{n-1} x^{n-1} + ..+ B_1 x + B_0)\, sin\,\beta x$

Notes : r is the smallest non negative integers to ensure no alike terms between $y_p(x)$ and $y_h(x)$.

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Table 6 : Variation of parameters method.

Homogeneous solution, $y_h(x) = Ay_1 + By_2$

Wronskian function, $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_2 y_1'$

$u_1 = -\int \frac{y_2 f(x)}{aW} dx + A$ $\qquad$ $u_2 = \int \frac{y_1 f(x)}{aW} dx + B$

General solution, $y(x) = u_1 y_1 + u_2 y_2$

Table 7 : Trigonometry Identities

$$\sin^2 t + \cos^2 t = 1$$

$$\sin^2 t = \frac{1}{2}(1 - \cos 2t)$$

$$\cos^2 t = \frac{1}{2}(1 + \cos 2t)$$

Table 8 : Partial Fraction

$$\frac{a}{(s+b)(s^2+c)} = \frac{A}{(s+b)} + \frac{Bs+C}{(s^2+c)}$$

$$\frac{a}{s(s-b)(s-c)} = \frac{A}{s} + \frac{B}{s-b} + \frac{C}{s-c}$$

$$\frac{a}{(s+b)(s-c)} = \frac{A}{(s+b)} + \frac{B}{(s-c)}$$