CONFIDENTIAL

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2014/2015

COURSE NAME	:	CALCULUS
COURSE CODE	:	DAS 20803
PROGRAMME	:	2 DAU
EXAMINATION DATE	:	JUNE 2015 / JULY 2015
DURATION	:	3 HOURS
INSTRUCTION	:	A) ANSWER **ALL** QUESTIONS IN **PART A** B) ANSWER **THREE (3)** QUESTIONS ONLY IN **PART B**

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

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PART A

Q1 (a) Evaluate the following integrations using appropriate integration techniques.

(i) $\int\left(\frac{1}{\sqrt{x}}+x\right)dx$

(2 marks)

(ii) $\int\cos^3 2t\sin 2t\,dt$

(3 marks)

(iii) $\int xe^{2x}\,dx$

(4 marks)

(iv) $\int\frac{5-x}{(x+1)(x-2)}dx$

(4 marks)

(b) Using Trapezoidal rule, find the value for $\int_1^4\frac{1}{\sqrt{x+3}}dx$ by taking $h=0.5$.

(7 marks)

Q2 (a) Given two functions, $y=x^2-4x$, and $y=16-x^2$.

(i) Find the coordinate of the points at which both functions intersect.

(4 marks)

(ii) Sketch the graphs of the functions on the same axis.

(3 marks)

(iii) Find the area of the region enclosed by both functions.

(4 marks)

(b) (i) Sketch the graph of a region enclosed by $y=\sqrt{x+1}$, $x=0$, $y=0$ and $x=3$.

(2 marks)

(ii) Use cylindrical shells to find the volume of the solid that is generated when the area of the region is revolved about the y-axis.

(7 marks)

PART B

Q3 (a) The function $f(x)$ is given by

$$f(x)=\begin{cases} x^2 & x<2 \\ 4 & 2\le x<3 \\ 10-2x & 3\le x<6 \end{cases}$$

(i) Sketch the graph of $f(x)$.

(3 marks)

(ii) Write the domain and range of $f(x)$.

(2 marks)

(iii) Calculate the value of $f(x)$ when $x=1$ and $x=5$

(2 marks)

(b) Given $f(x)=x+2$, $g(x)=\dfrac{x}{3}-1$ and $h(x)=\dfrac{x}{2}$. Calculate the composite function of

(i) $f\circ h$

(3 marks)

(ii) $f\circ g\circ h$

(4 marks)

(c) If $g(x)=\dfrac{-k+x}{3}$, find

(i) The inverse function, $g^{-1}(x)$.

(3 marks)

(ii) If $g^{-1}(1)=4$, find the value of k

(3 marks)

Q4 (a) Evaluate the following limits using L'Hospital's Rule.

(i) $\lim_{x\to 4}\left(\dfrac{x-4}{4-x}\right)$

(3 marks)

(ii) $\lim_{x\to\infty}\dfrac{x^3+3}{2x^3+4x+1}$

(5 marks)

(iii) $\lim_{x\to 1}\dfrac{x^2+12x-13}{x-1}$

(3 marks)

(iv) $\lim_{x\to 0}\dfrac{1-\cos x}{x^2}$

(4 marks)

(b) Determine the value of a so that

$$f(x)=\begin{cases} x^2-1, & x<3 \\ 2ax, & x\geq 3 \end{cases}$$

is continuous at the point $x=3$.

(5 marks)

Q5 (a) Find $\dfrac{dy}{dx}$ of

(i) $y(x) = 2\sqrt{x} + 4x^2 - \dfrac{5}{x^4}$

(3 marks)

(ii) $y(x) = 5x\sin 2x$

(3 marks)

(iii) $y(x) = \ln(3x^2+2)$

(3 marks)

(iv) $y(x) = \dfrac{4x^2-5x}{x+3}$

(4 marks)

(b) Using implicit differentiation, find $\dfrac{dy}{dx}$ for $x^2+xy-2y^3=25$

(4 marks)

(c) Given function $y=3x^3+9x$. Find y'' and y'''.

(3 marks)

Q6 (a) A balloon is filled with air at the rate of 20 cm^3/minute until a sphere is formed. Find the rate of change of the radius when the radius is 4 cm. Volume of sphere, $V = \frac{4}{3}\pi r^3$.

(4 marks)

(b) The volume of a cube decreases at a rate of 500 m^3/min. What is the rate of change of the side length when the side length are 12 meter?. Volume of a cube, $V = x^3$.

(4 marks)

(c) Given a curve of $y(x) = 2x^3 - 5x^2 - 4x + 4$.

(i) Find all the critical points.

(4 marks)

(ii) Find the value of x when $y''(x) = 0$

(2 marks)

(iii) Determine the maximum and minimum points.

(6 marks)

Q7 (a) Evaluate the following integral using substitution method.

$$\int_0^1 \frac{2x+5}{(x^2+5x+3)^3}\, dx$$

(6 marks)

(b) If $y = Ax + Bx^2$, where A and B are constants, show that

$$x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$$

(5 marks)

(c) Find the arc length of the curve $y = x^{\frac{3}{2}}$ from $x = 0$ to $x = 5$.

(9 marks)

– END OF QUESTION –

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Formula

Table 1 : Differentiation	
$\frac{d}{dx}[c]=0$	$\frac{d}{dx}[\sin x]=\cos x$
$\frac{d}{dx}[x^n]=nx^{n-1}$	$\frac{d}{dx}[\cos x]=-\sin x$
$\frac{d}{dx}[cu]=c\frac{du}{dx}$	$\frac{d}{dx}[\tan x]=\sec^2 x$
$\frac{d}{dx}[u\pm v]=\frac{du}{dx}\pm\frac{dv}{dx}$	$\frac{d}{dx}[\sec x]=\sec x\tan x$
$\frac{d}{dx}[uv]=u\frac{dv}{dx}+v\frac{du}{dx}$	$\frac{d}{dx}[\cot x]=-\csc^2 x$
$\frac{d}{dx}\left[\frac{u}{v}\right]=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$	$\frac{d}{dx}[\csc x]=-\csc x\cot x$
$\frac{d}{dx}[\ln\lvert x\rvert]=\frac{1}{x}$	$\frac{d}{dx}[\sin^{-1}x]=\frac{1}{\sqrt{1-x^2}}$
$\frac{d}{dx}\left[e^{nx}\right]=ne^{nx}$	$\frac{d}{dx}[\cos^{-1}x]=\frac{-1}{\sqrt{1-x^2}}$
$\frac{d}{dx}\left[\sec^{-1}x\right]=\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	$\frac{d}{dx}\left[\tan^{-1}x\right]=\frac{1}{1+x^2}$

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Table 2 : Integration

$\int c\ f(x)dx = c\ F(x) + C$	$\int \tan x\ dx = \ln\lvert \sec x \rvert + C$
$\int [f(x) \pm g(x)]dx = F(x) \pm G(x) + C$	$\int \sec^2 x\ dx = \tan x + C$
$\int x^r\ dx = \dfrac{x^{r+1}}{r+1} + C, (r \neq -1)$	$\int \csc^2 x\ dx = -\cot x + C$
$\int u dv = uv - \int v du$	$\int \csc x\ dx = -\ln\lvert \csc x + \cot x \rvert + C$
$\int \cos x\ dx = \sin x + C$	$\int \sec x \tan x\ dx = \sec x + C$
$\int \sin x\ dx = -\cos x + C$	$\int \csc x \cot x\ dx = -\csc x + C$
$\int e^{nx}\ dx = \dfrac{1}{n} e^{nx} + C$	$\int \sec x\ dx = \ln\lvert \sec x + \tan x \rvert + C$
$\int \dfrac{1}{ax+b} dx = \dfrac{1}{a} \ln\lvert ax+b \rvert + C$	

Table 3 : Partial Fractions

$\dfrac{a}{(s+b)(s-c)} = \dfrac{A}{(s+b)} + \dfrac{B}{(s-c)}$
$\dfrac{a}{s(s-b)(s-c)} = \dfrac{A}{s} + \dfrac{B}{s-b} + \dfrac{C}{s-c}$
$\dfrac{a}{(s+b)(s^2+c)} = \dfrac{A}{(s+b)} + \dfrac{Bs+C}{(s^2+c)}$

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Area of Region

$$A=\int_a^b [f(x)-g(x)]\,dx \quad \text{or} \quad A=\int_c^d [w(y)-v(y)]\,dy$$

Volume Cylindrical Shells

$$V=\int_a^b 2\pi x\, f(x)\,dx \quad \text{or} \quad V=\int_c^d 2\pi y\, f(y)\,dy$$

Arc Length

$$L=\int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx \quad \text{or} \quad L=\int_c^d \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

Area of Surface

$$S=\int_a^b 2\pi y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx \quad \text{or} \quad S=\int_c^d 2\pi x\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

Simpson's Rule

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[(f_0+f_n)+4\sum_{\substack{i=1\\ i\ \text{odd}}}^{n-1} f_i+2\sum_{\substack{i=2\\ i\ \text{even}}}^{n-2} f_i\right]; \quad n=\frac{b-a}{h}$$

Trapezoidal Rule

$$\int_a^b f(x)\,dx \approx \frac{h}{2}\left[(f_0+f_n)+2\sum_{i=1}^{n-1} f_i\right]; \quad n=\frac{b-a}{h}$$

Improper Integral

$$\int_a^{+\infty} f(x)\,dx=\lim_{C\to+\infty}\int_a^C f(x)\,dx \qquad \int_{-\infty}^b f(x)\,dx=\lim_{G\to-\infty}\int_G^b f(x)\,dx$$