Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2023/2024

COURSE NAME	:	DISCRETE MATHEMATICS
COURSE CODE	:	DAT 10203
PROGRAMME CODE	:	DAT
EXAMINATION DATE	:	JULY 2024
DURATION	:	3 HOURS
INSTRUCTIONS	:	1. ANSWER ALL QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA ☐ Open book ☒ Closed book 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES.

Q1 (a) If A = {1, {2, 3}, 4}, identify each of the following as **TRUE** or **FALSE**:

(i) $3 \in A$

(ii) $\{1, 4\} \subset A$

(iii) $\{2, 3\} \in A$

(iv) $\{4\} \in A$

(v) $1 \notin A$

(5 marks)

(b) Let A = {1}, B = {1, a, 2, b, c}, C = {b, c}, D = {a, b}, and E = {1, a, 2, b, c, d}. For each of the following, replace the symbol ☐ with $\subseteq$ or $\nsubseteq$ to give a **TRUE** statement.

(i) A ☐ B

(ii) Ø ☐ A

(iii) B ☐ C

(iv) C ☐ E

(v) D ☐ C

(5 marks)

(c) ITeCH Sdn Bhd wants to hire 25 programmers for systems programming jobs and 40 for applications programming. How many programmers must the company hire if they expect ten of them to perform jobs of both types?

(5 marks)

Q2 (a) Find the domain and range for the following relation R:

A = {daisy, rose, violet, daffodil, peony}

B = {red, white, purple, yellow, blue, pink, orange}

R = {(daisy, red), (violet, pink), (rose, purple), (daffodil, white)}

(4 marks)

(b) Let X = {1, 2, 3, 4}. Determine whether the following relation R whose matrix as shown in **Figure Q2.1** is reflexive, irreflexive, symmetric, antisymmetric, or asymmetric.

$$M_R = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix}$$

Figure Q2.1 M_R Matrix for X.

(6 marks)

(c) If $A = Z^+$, and R is the relation defined by aRb if and only if $2a \leq b + 1$, which of the following ordered pairs belong to R? Show calculations for each case and indicate whether it belongs to R with **YES** or **NO**.

(i) (2, 2)

(ii) (3, 2)

(iii) (6, 15)

(iv) (1, 1)

(v) (15, 6)

(5 marks)

Q3 (a) Let $A = B = C = \mathbb{R}$, and let $f: A \rightarrow B$, $g: B \rightarrow C$ is defined by $f(a) = a - 1$ and $g(b) = b^2$. Find:

(i) $(f \circ g)(2)$

(ii) $(g \circ f)(2)$

(iii) $(g \circ f)(x)$

(iv) $(f \circ g)(x)$

(v) $(f \circ f)(y)$

(8 marks)

(b) Prepare the truth table of each proposition if p and r are true and q is false.

(i) $\sim p \wedge (q \vee r)$

(ii) $p \wedge (\sim (q \vee \sim r))$

(iii) $(r \wedge \sim q) \vee (p \vee r)$

(iv) $(q \wedge r) \wedge (p \vee \sim r)$

(7 marks)

(c) Let A = "Andrew is Sarawakian" and F = "Fazulika is Sabahan". Translate the following English sentence into logical expressions.

(i) Andrew isn't Sarawakian.

(ii) Andrew is Sarawakian while Fazulika is Sabahan.

(iii) If Andrew is Sarawakian then Fazulika is not Sabahan.

(iv) Andrew is Sarawakian or if Andrew isn't Sarawakian then Fazulika is Sabahan.

(v) Either Andrew is Sarawakian and Fazulika is Sabahan or, neither Andrew is Sarawakian nor Fazulika is Sabahan.

(10 marks)

Q4 (a) State whether the argument given is valid or invalid. If it is valid, identify the rule of inference and the tautology or tautologies on which it is based. You may refer to **Figure APPENDIX A.1** for the Rule of Inference.

(i) If I drive to work, then I will arrive tired.
I am not tired when I arrive at work.
$\therefore$ I do not drive to work.

(ii) If I drive to work, then I will arrive tired.
I do not drive to work.
$\therefore$ I will not arrive tired.

(10 marks)

(b) Calculate the value of $(61^2 - 39^2) \div (51^2 - 49^2)$ without using the calculator.

(5 marks)

(c) Based on **Figure Q4.1,** identify the following:

(i) Sets of edges and vertices.

(ii) The degree of each vertex and the total degrees of the graph.

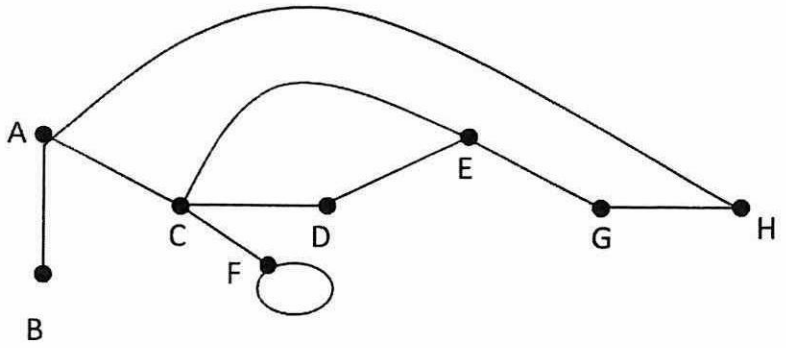

Figure Q4.1 The graph with its vertices and edges.

(10 marks)

Q5 (a) Based on a binary tree in **Figure Q5.1**, find:

(i) Preorder traversal

(ii) Inorder traversal

(iii) Postorder traversal

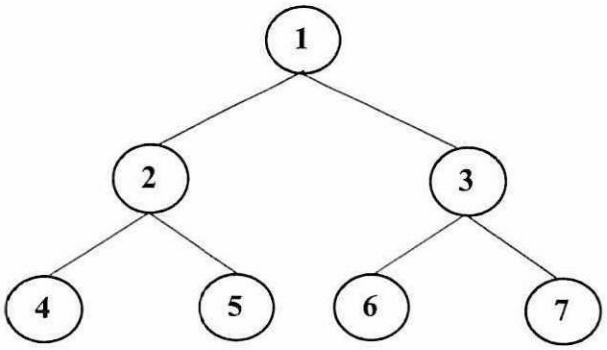

Figure Q5.1 The binary tree.

(6 marks)

(b) Assume that each of the expressions in **Table Q5.1** gives the processing time *T (n)* spent by an algorithm for solving a problem of size *n*. Select the dominant term(s) with the steepest increase in n and specify the lowest Big-Oh complexity of each algorithm.

Table Q5.1 The time complexity and Big-Oh notation.

Expression	Dominant term(s)	O (. . .)
$5 + 0.001n^3 + 0.025n$		
$500n + 100n^{1.5} + 50n \log_{10} n$		
$0.3n + 5n^{1.5} + 2.5 \cdot n^{1.75}$		
$100n + 0.01n^2$		
$0.01n + 100n^2$		
$2n + n^{0.5} + 0.5n^{1.25}$		
$100n \log_3 n + n^3 + 100n$		

(14 marks)

- END OF QUESTIONS -

APPENDIX A

Rule of Inference	Tautology	Name
$\begin{array}{l} p \\ \underline{p \rightarrow q} \\ \therefore q \end{array}$	$(p \wedge (p \rightarrow q)) \rightarrow q$	Modus Ponens
$\begin{array}{l} \neg q \\ \underline{p \rightarrow q} \\ \therefore \neg p \end{array}$	$(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$	Modus Tollens
$\begin{array}{l} p \rightarrow q \\ \underline{q \rightarrow r} \\ \therefore p \rightarrow r \end{array}$	$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$	Hypothetical syllogism
$\begin{array}{l} \neg p \\ \underline{p \vee q} \\ \therefore q \end{array}$	$(\neg p \wedge (p \vee q)) \rightarrow q$	Disjunctive Syllogism
$\begin{array}{l} p \\ \hline \therefore (p \vee q) \end{array}$	$p \rightarrow (p \vee q)$	Addition
$\begin{array}{l} (p \wedge q) \rightarrow r \\ \hline \therefore p \rightarrow (q \rightarrow r) \end{array}$	$((p \wedge q) \rightarrow r) \rightarrow (p \rightarrow (q \rightarrow r))$	Exportation
$\begin{array}{l} p \vee q \\ \underline{\neg p \vee r} \\ \therefore q \vee r \end{array}$	$((p \vee q) \wedge (\neg p \vee r)) \rightarrow q \vee r$	Resolution

Figure APPENDIX A.1