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Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2023/2024

COURSE NAME	:	ENGINEERING MATHEMATICS II
COURSE CODE	:	DAC 12403
PROGRAMME CODE	:	DAA
EXAMINATION DATE	:	JULY 2024
DURATION	:	3 HOURS
INSTRUCTIONS	:	1. ANSWER ALL QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA ☐ Open book ☒ Closed book 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

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Q1 (a) Given a differential equation $\left(8xy+12x^2\right)dy-\left(4y^2+12x^2\right)dx=0$.

(i) Show that the differential equation above is a homogeneous equation.

(3 marks)

(ii) Then solve the equation.

(7 marks)

(b) Find the particular solution of the given linear equation:

$$(x-3)\frac{dy}{dx}+y=\ln x \quad ; \quad y(1)=3.$$

(10 marks)

Q2 (a) Solve the following second order differential equations:

(i) $y''+4y'+5y=0$.

(3 marks)

(ii) $y''-4y'+4y=0$ with initial condition $y(0)=1$ and $y'(0)=2$.

(7 marks)

(b) By using the method of variation of parameters, determine the general solution of:

$$y''+4y=\operatorname{cosec}(2x).$$

(10 marks)

Q3 Find the Laplace Transform for the following functions:

(a) $f(t)=\cos(\alpha t+\beta)$.

(3 marks)

(b) $f(t)=\frac{3}{2}e^{-\frac{1}{2}t}-\sinh\left(\frac{3}{2}t\right)+5t^5$.

(3 marks)

(c) $f(t)=\left(\frac{\pi}{2}-t\right)^2-\frac{5}{3}\cosh(4t)+e^{-\frac{\pi}{2}t}$.

(4 marks)

(d) $f(t)=e^{2t}\left(t(t+2)(t-4)\right)$ by using First Shift Theorem.

(4 marks)

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(e) $f(t)=t^2\sin 5t$ by using Multiplication by t^n.

(6 marks)

Q4 (a) Find the inverse Laplace transform for the following expressions:

(i) $\dfrac{4s}{s^2+9}$.

(1 mark)

(ii) $\dfrac{2s+9}{s^2-7}$.

(3 marks)

(iii) $\dfrac{3-s}{s^4}$.

(4 marks)

(b) Determine the inverse Laplace transform of $\dfrac{2s+1}{(s+2)(s^2+3)}$ by using partial fraction.

(7 marks)

(c) By using Convolution theorem, evaluate the inverse Laplace transform of $\dfrac{3}{2(s+5)(s-2)}$.

(5 marks)

Q5 Solve the following differential equations by using Laplace transform.

(a) $2\dfrac{d^2y}{dt^2}+\dfrac{dy}{dt}=7t$ where $y(0)=1$ and $y'(0)=5$.

(10 marks)

(b) $\dfrac{dy}{dt}-9y=3$ where $y(\pi)=-\dfrac{1}{3}$.

(10 marks)

- END OF QUESTIONS -

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APPENDIX A

Formula

Table 1: Laplace and Inverse Laplace Transforms

$$L\{f(t)\} = \int_0^\infty f(t)e^{-st}dt = F(s)$$

$f(t)$	$F(s)$
k	$\frac{k}{s}$
$t^n, n = 1, 2, \ldots$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$

First Shift Theorem

$e^{at}f(t)$	$F(s-a)$

Multiply with t^n

$t^n f(t), n = 1, 2, \ldots$	$(-1)^n \frac{d^n F(s)}{ds^n}$

Convolution Property:

If $F(s) = G(s)H(s)$, then $L^{-1}G(s) = g(t)$ and $L^{-1}H(s) = h(t)$.

$$f(t) = L^{-1}F(s) = L^{-1}\left[G(s)H(s)\right] = \int_0^t g(\tau)h(t-\tau)d\tau \text{ or } \int_0^t h(\tau)g(t-\tau)d\tau$$

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Initial Value Problem
$L\{y(t)\}=Y(s)$
$L\{y'(t)\}=sY(s)-y(0)$
$L\{y''(t)\}=s^2Y(s)-sy(0)-y'(0)$

Table 2: Differentiation

$\frac{d}{dx}[c]=0$	$\frac{d}{dx}[\sin u]=\cos u\left(\frac{du}{dx}\right)$
$\frac{d}{dx}[x^n]=nx^{n-1}$	$\frac{d}{dx}[\cos u]=-\sin u\left(\frac{du}{dx}\right)$
$\frac{d}{dx}[cu]=c\frac{du}{dx}$	$\frac{d}{dx}[\ln\|u\|]=\frac{1}{u}\left(\frac{du}{dx}\right)$
$\frac{d}{dx}[u\pm v]=\frac{du}{dx}\pm\frac{dv}{dx}$	$\frac{d}{dx}\left[e^u\right]=e^u\left(\frac{du}{dx}\right)$
$\frac{d}{dx}[uv]=u\frac{dv}{dx}+v\frac{du}{dx}$	$\frac{d}{dx}[a^u]=a^u\ln a\left(\frac{du}{dx}\right)$
$\frac{d}{dx}\left[\frac{u}{v}\right]=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$	$\frac{d}{dx}[\tan u]=\sec^2 u\left(\frac{du}{dx}\right)$
$\frac{d}{dx}[\log_a u]=\frac{1}{u}\log_b e\left(\frac{du}{dx}\right)$	$\frac{d}{dx}[\sec u]=\sec u\tan u\left(\frac{du}{dx}\right)$

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Table 3: Integration

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$	$\int \cos ax \, dx = \frac{1}{a} \sin ax + C$
$\int \frac{1}{x} dx = \ln\lvert x\rvert + C$	$\int \tan ax \, dx = \frac{1}{a} \ln\lvert \sec ax\rvert + C$
$\int \frac{1}{a+bx} dx = \frac{1}{b} \ln\lvert a+bx\rvert + C$	$\int udv \, dx = uv - \int vdu$
$\int \sin ax \, dx = -\frac{1}{a} \cos ax + C$	$\int e^{ax} dx = \frac{1}{a} e^{ax} + C$

Table 4: Characteristic Equation and General Solution

Homogeneous Differential Equation: $ay'' + by' + cy = 0$

Characteristics Equation: $am^2 + bm + c = 0$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Case	Roots of Characteristics Equation	General Solution
1	Real and Distinct: $m_1 \neq m_2$	$y_h(x) = Ae^{m_1 x} + Be^{m_2 x}$
2	Real and Equal: $m_1 = m_2 = m$	$y_h(x) = (A + Bx)e^{mx}$
3	Complex Roots: $m = \alpha \pm i\beta$	$y_h(x) = e^{\alpha x}(A\cos \beta x + B\sin \beta x)$

Table 5: Variation of Parameters Method

Homogeneous solution, $y_h(x) = Ay_1 + By_2$
Wronskian function, $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_2 y_1'$
$u_1 = -\int \frac{y_2 f(x)}{aW} dx + A$ $\quad$ $u_2 = \int \frac{y_1 f(x)}{aW} dx + B$
General solution, $y(x) = u_1 y_1 + u_2 y_2$

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Table 6: First Order Ordinary Differential Equation

$y = vx \rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$
$\rho = e^{\int P(x)dx}$ $y = \frac{1}{\rho}\left[\int \rho \times Q(x)\,dx\right]$

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