Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2023/2024

COURSE NAME	:	ACTUARIAL MATHEMATICS II
COURSE CODE	:	BWA 31503
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	JULY 2024
DURATION	:	2 HOURS 30 MINUTES
INSTRUCTIONS	:	1. ANSWER ALL QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA ☐ Open book ☒ Closed book 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

Q1 A multiple-decrement life table summarizes the life experience of a cohort in which membership can be terminated by two or more attrition factors.

(a) Given that decrement may be due to death,1, disability, 2, or retirement, 3, use absolute rates in **Table Q1.1** to construct a multiple decrement table as shown in **Table Q1.2**.

Table Q1.1: Absolute Rates

Age, x	$q_x'^{(1)}$	$q_x'^{(2)}$	$q_x'^{(3)}$
62	0.020	0.030	0.200
63	0.022	0.034	0.100
64	0.028	0.040	0.120

Table Q1.2: Multiple Decrement Table

Age, x	$p_x^{(\tau)}$	$q_x^{(1)}$	$q_x^{(2)}$	$q_x^{(3)}$
62				
63				
64				

(12 marks)

(b) You are given the following portion of a double-decrement table (**Table Q1.3).**

Table Q1.3: Double-Decrement Table

Age, x	$l_x^{(\tau)}$	$d_x^{(1)}$	$d_x^{(2)}$
50	1100	100	300
51	700	50	180
52	470	40	110
53	320		

A 2-years insurance contract on a 50-years-old man provides for benefits paid at the end of the year of failure if this occurs within 2 years. The benefit payable is RM 10,000 if failure is from cause 1, or RM 20,000 if it is from cause 2. If the interest rate is a constant 50% per year, compute the net single premium of the insurance.

(8 marks)

Q2 A Lexis diagram (**Figure Q2.1**) provides a convenient way of showing the relationship between periods and cohorts. Demographic events can be viewed either by calendar time, age or cohort.

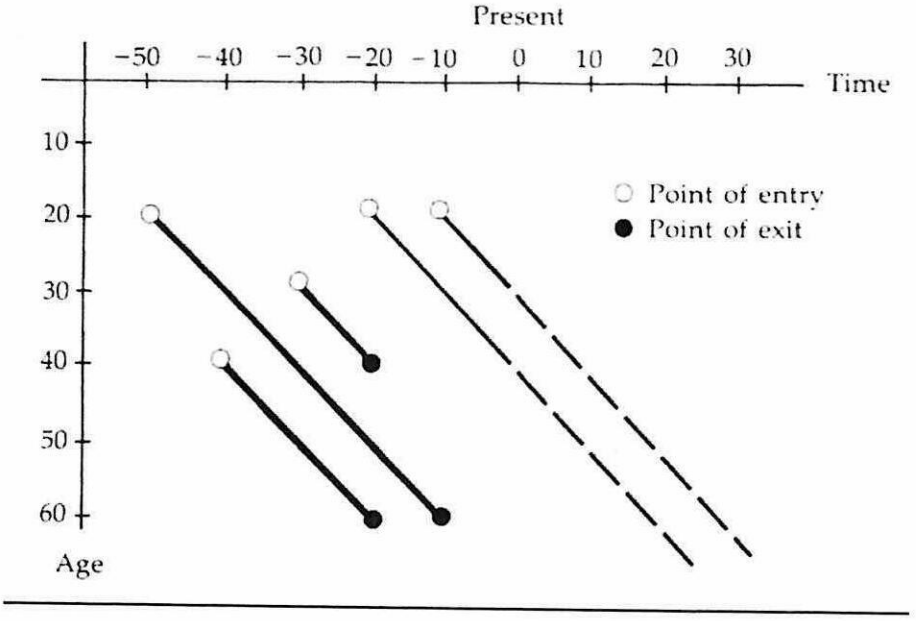

Figure Q2.1: Lexis Diagram

(a) Using the Lexis diagram in **Figure Q2.1**, compute

(i) the average age of employees at time -30;

(2 marks)

(ii) the number of employees who have attained age 55 in the history of this workforce;

(1 mark)

(iii) of the employees at time -30, the number who have attained or will attain age 55 while in the workforce.

(1 mark)

(b) A number of people between the ages of 20 and 50 at time 0 will die at an age less than 80.

(i) Illustrate a Lexis Diagram depicting this situation.

(6 marks)

(ii) Generate the integrals that will calculate the number of those between ages 20 and 50 at time 0 that will die at an age less than 80.

(10 marks)

Q3 A pension plan provides for an annual pension of 1.8% of the final 3-year average salary h, times the number of years of service, for retirement at ages 63 – 65, but with a reduction of 5% per year should retirement occur before 65. An employee, age 45, who began work at 35, is comparing their income for retirement at 63, 64 and 65. With RM 70,000 current annual salary, he expects a 4% increase in salary each year.

(a) Compare the final 3-year average salary by finding h if the employee chooses to retire at 63, 64, or 65.

(6 marks)

(b) Outline the pension incomes that would be received by the employee if he chooses to retire at 63, 64 or 65.

(6 marks)

(c) Identify the redemption ratio if the employee chooses to retire at 65.

(3 marks)

(d) Compute the ratio of the pension income for retirement in **TWENTY (20)** years to that for retirement in **EIGHTEEN (18)** years.

(2 marks)

(e) Formulate the actuarial present value of the pension incomes.

(3 marks)

Q4 **Tables Q4.1**, **Q4.2** and **Q4.3** specify the expenses and withdrawal benefits of an endowment insurance.

Table Q4.1: Description

Plan of insurance	3-year annual premium endowment insurance, issued to (x) with level benefits and premiums
Payment basis	Fully discrete
Mortality	$q_x = 0.1$, $q_{x+1} = 0.1111$, $q_{x+2} = 0.5$
Interest	Annual effective rate of $i = 15\%$
Amount of insurance	1,000
Expenses	
a. Timing	Paid at the beginning of each policy year
b. Amount	(as given in **Table Q4.2**)

Table Q4.2: Amount of Expenses

Type of Expense	**First Year**		**Renewal Year**	
	Percentage of Premium (%)	**Constant**	**Percentage of Premium (%)**	**Constant**
Sales commission	10	-	2	-
General expense	4	3	-	1
Taxes, licenses and fees	2	-	2	-
Policy maintenance	2	1	2	1
Issue and classification	2	4	-	-
Total	20	6	6	2

Table Q4.3: Withdrawal benefits

Withdrawal benefits	$b_{x+1}^{(2)} = 227.73$; $b_{x+2}^{(2)} = 564.41$
Multiple Decrement Probabilities	$q_x^{(1)} = 0.1$; $q_{x+1}^{(1)} = 0.1111$ $q_x^{(2)} = 0.1$; $q_{x+1}^{(2)} = 0.1111$

(a) Calculate asset shares for the first and second year if $G = 342.96$.

(6 marks)

(b) The following experience adjustments are then made to **Q4(a)**:

$${}_0F =_0 AS$$
$${}_1F =_1 AS$$
$${}_2F =_2 AS$$
$$\hat{i}_1 = 0.15,\ \hat{i}_2 = 0.16$$
$$\hat{q}_x^{(1)} = 0.085,\ \hat{q}_x^{(2)} = 0.200$$
$$\hat{q}_{x+1}^{(1)} = 0.090,\ \hat{q}_{x+1}^{(2)} = 0.100$$
$$c_h = \hat{c}_h,\ \text{for } h = 0,1,2$$
$$\hat{e}_0 = 10,\ \hat{e}_1 = 1$$

Break down the dividends ${}_1D$ and ${}_2D$ for two years with G = 342.96.

(14 marks)

- END OF QUESTIONS -

APPENDIX A

LIST OF FORMULAS

$$d_x^{(\tau)} = \sum_{j=1}^{m} d_x^{(j)}. \qquad \ell_{x+1}^{(\tau)} = \ell_x^{(\tau)} - d_x^{(\tau)}. \qquad q_x^{(j)} = \frac{d_x^{(j)}}{\ell_x^{(\tau)}}$$

$$_kp_x^{(\tau)} = \frac{\ell_{x+k}^{(\tau)}}{\ell_x^{(\tau)}} \qquad _nq_x^{(j)} = \frac{\sum_{k=0}^{n-1} d_{x+k}^{(j)}}{\ell_x^{(\tau)}} \qquad \frac{d_{x+k}^{(j)}}{\ell_x^{(\tau)}} = {}_kp_x^{(\tau)} q_{x+k}^{(j)}$$

$$\ell_x^{(j)} = \sum_{k=0}^{\omega-x-1} d_{x+k}^{(j)}. \qquad \ell_x^{(\tau)} = \sum_{j=1}^{m} \ell_x^{(j)}. \qquad d_x^{(j)} = \ell_x^{(j)} - \ell_{x+1}^{(j)}.$$

$$_nq_x^{(j)} = \frac{\ell_x^{(j)} - \ell_{x+n}^{(j)}}{\ell_x^{(\tau)}}. \qquad p_x'^{(j)} = p_x^{(\tau)\left(\frac{q_x^{(j)}}{q_x^{(\tau)}}\right)} \qquad q_x^{(j)} = \frac{\log p_x'^{(j)}}{\log p_x^{(\tau)}} q_x^{(\tau)}.$$

$$p^{(\tau)} = p'^{(1)} p'^{(2)}. \qquad A_{x:\overline{n}|}^{1} = \sum_{k=0}^{n-1} v^{k+1}\ {}_{k|}q_x \qquad {}_{t|u}q_x = {}_tp_x - {}_{t+u}p_x$$

$$A_x = \sum_{k=0}^{\infty} v^{k+1}\ {}_{k|}q_x \qquad \mu(x,u) = -\frac{1}{l(x,u)} \frac{\partial}{\partial x} l(x,u)$$

$$p_{x+h-1}^{(\tau)}\left({}_hAS\right) = \left[{}_{h-1}AS + G\left(1-c_{h-1}\right) - e_{h-1}\right]\left(1+i\right) - b_h q_{x+h-1}^{(1)} - {}_hCV\, q_{x+h-1}^{(2)}$$

$$\begin{aligned} {}_{h+1}D = {} & \left({}_hF + G\right)\left(\hat{i}_{h+1} - i\right) \\ & + \left[\left(Gc_h + e_h\right)\left(1+i\right) - \left(G\hat{c}_h + \hat{e}_h\right)\left(1+\hat{i}_{h+1}\right)\right] \\ & + \left(b_{h+1} - {}_{h+1}F\right)\left(q_{x+h}^{(1)} - \hat{q}_{x+h}^{(1)}\right) \\ & + \left({}_{h+1}CV - {}_{h+1}F\right)\left(q_{x+h}^{(2)} - \hat{q}_{x+h}^{(2)}\right) \end{aligned}$$