Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2023/2024

COURSE NAME	:	FUZZY SET THEORY AND APPLICATIONS
COURSE CODE	:	BWA 11203 / BWA 21203
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	JULY 2024
DURATION	:	3 HOURS
INSTRUCTIONS	:	1. ANSWER ALL QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA ☐ Open book ☒ Closed book 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **FOUR (4)** PAGES

Q1 A fuzzy set A is said to be convex if the universal set X is a vector space and $A(\lambda x + (1-\lambda)y) \geq \min\{A(x), A(y)\}$ for all $x, y \in X$, and $\lambda \in [0,1]$. Show that fuzzy set A is convex if and only if the crisp set $A_\alpha = \{x \in X : A(x) \geq \alpha\}$ is convex for each $\alpha \in [0,1]$.

(5 marks)

Q2 Consider the fuzzy sets A and B are defined as universe, $X = \{0,1,2,3\}$ by the membership functions

$$\mu_A(x) = \frac{2}{x+3}, \qquad \mu_B(x) = \frac{4x}{2(x+5)}.$$

Examine the intervals along x-axis corresponding to the α- cut sets for each fuzzy sets A and B for the following values of α, $\alpha = 0.2, 0.5,$ and 0.6.

(5 marks)

Q3 Let A and B be fuzzy set defined on the universal set $X = Z$ whose membership functions are given by

$$A(x) = \frac{0.5}{-1} + \frac{1}{0} + \frac{0.5}{1},$$

and

$$B(x) = \frac{0.5}{2} + \frac{1}{3} + \frac{0.5}{4}.$$

Let a function $f : X \times X \to X$ be defined for all $x_1, x_2 \in X$ by $f(x_1, x_2) = x_1 \cdot x_2$. Calculate $f(A, B)$.

(7 marks)

Q4 Given function

$$c(a) = \frac{\gamma^2(1-a)}{a + \gamma^2(1-a)}, \qquad \forall a \in [0,1], \gamma > 0.$$

(a) Show that the function is a fuzzy complement.

(4 marks)

(b) Plot the function for some values of γ.

(2 marks)

(c) Demonstrate that the increasing generator $g_\lambda(a) = \frac{a}{\gamma + (1-\gamma)a}, \gamma > 0$ is a class of fuzzy complement. [**Hint**: $c_\lambda(a) = g_\lambda^{-1}(g_\lambda(1) - g_\lambda(a))$]

(5 marks)

Q5 Let $i(a,b) = ab$ and let $c_\lambda(a) = \frac{1-a}{1+\lambda a}$, $\lambda \geq 0$ be the Sugeno's class of fuzzy complements, obtain the t −conorm u such that $\langle i, u, c \rangle$ is a dual triple.

(7 marks)

Q6 There are two inputs, I_1 and I_2, and an output O of a process. Developing a fuzzy logic controller (FLC) based on the Mamdani approach is required. The inputs and output are expressed using three linguistic terms, namely L (low), M (medium) and H (high). The membership function distributions of the above inputs and output are shown in **Figure Q6.1**. The rule base of the fuzzy logic controller is shown in the **Table Q6.1**.

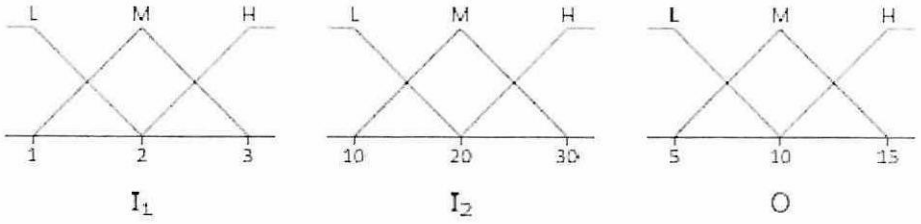

Figure Q6.1. Membership Function Distributions

Table Q6.1. Rules

		I_2		
		L	**M**	**H**
I_1	**L**	L	L	M
	M	L	M	H
	H	M	H	H

Suppose at any instant, inputs to the fuzzy logic controller are $I_1 = 1.5$ and $I_2 = 25$.

(a) Identify the fuzzified values of the input.

(8 marks)

(b) Compute rule strengths of the rules corresponding to the given inputs.

(5 marks)

(c) Decide the fuzzy outputs for the given inputs.

(7 marks)

(d) Defuzzified the output using Centroid method.

(5 marks)

Q7 In the field of computer networking there is an imprecise relationship between the level of use of a network communication bandwidth and the latency experienced in peer-to-peer communication. Let X be a fuzzy set of use levels (in terms of the percentage of full bandwidth used) and Y be a fuzzy set of latencies (in milliseconds) with the following membership function:

$$X = \left\{\frac{0}{5} + \frac{0.1}{30} + \frac{0.3}{50} + \frac{0.8}{100} + \frac{1.0}{300}\right\},$$

$$Y = \left\{\frac{0.7}{2} + \frac{0.8}{4} + \frac{0.2}{8} + \frac{0.1}{10} + \frac{0.7}{12}\right\}.$$

(a) Compute the Cartesian product represented by the relation $R = X \times Y$.

(10 marks)

(b) Now, suppose we have second fuzzy set of bandwidth usage given by

$$Z = \left\{\frac{1.0}{5} + \frac{0.8}{30} + \frac{0.1}{50} + \frac{0.2}{100} + \frac{0}{300}\right\}.$$

Find a relation between a Z and previously determines relation of **Q7(a)** ($S = Z \circ R$)

(i) using max-min composition.

(5 marks)

(ii) using max-product composition.

(5 marks)

- END OF QUESTIONS -