UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2023/2024

COURSE NAME	:	MATHEMATICAL MODELLING
COURSE CODE	:	BWA 30803
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	JULY 2024
DURATION	:	3 HOURS
INSTRUCTIONS	:	1. ANSWER ALL QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA ☐ Open book ☒ Closed book 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **FOUR (4)** PAGES

Q1 Consider the system of differential equations

$$x' = x + x^2 - 2xy,$$
$$y' = -y + xy.$$

(a) Determine the steady state(s) / fixed point(s) of the system.

(5 marks)

(b) Analyse the system using linear stability analysis and conclude the stability of each steady state / fixed point from the eigenvalues found.

(10 marks)

(c) From the results in **Q1(b)**, sketch the phase plane y versus x.

(6 marks)

Q2 Suppose in a chemical reaction of two substances, M_1 and M_2, react in equal amounts to form a compound M_3. Let $C(t)$ be the concentration of the compound M_3 at time t, which satisfies the differential equation

$$\frac{dC}{dt} = r(a - C)(b - C),$$

where r is a positive constant, a and b are initial concentrations of M_1 and M_2 at time $t = 0$.

(a) Obtain the concentration of M_3 as a function of time for $t > 0$, assuming $C(0) = 0$.

(4 marks)

(b) Determine the limiting concentration when $a = 9$ and $b = 14$.

(3 marks)

Q3 A mathematical model for epidemics consisting of susceptible (S), infected (I) and removals (R) is given by

$$\frac{dS}{dt} = -\beta S^2 I,$$
$$\frac{dI}{dt} = \beta S^2 I - \gamma I,$$
$$\frac{dR}{dt} = \gamma I,$$

where β and γ are positive constants.

(a) Determine the threshold density of susceptible.

(3 marks)

(b) Show that

$$\frac{dR}{dt} = \gamma\left(n - R - \frac{S_0}{1 + \frac{R S_0 \beta}{\gamma}}\right).$$

(8 marks)

Q4 Suppose a single infected individual migrates into a community containing n individuals susceptible to a disease. The infected individual spreads the disease to all susceptible. If $S(t)$ be the number of susceptible at time t, then

$$\frac{dS}{dt} = -r\,S\,(n+1-S) \quad \text{with} \quad S(0) = n,$$

and r is a positive constant which measures the infection. Obtain the solution of this model and comment.

(7 marks)

Q5 The dynamical system that models the amount of alcohol in a person's body is given by

$$U_{n+1} = U_n - \frac{9U_n}{4.2 + U_n} + d\,,$$

where U_n is the number of grams of alcohol in the body at the beginning of hour n, and d is the constant amount consumed per hour.

(a) Find the equilibrium value, given that this person consumes 7 grams of alcohol per hour.

(3 marks)

(b) Determine the stability of the system.

(7 marks)

Q6 Consider the price model

$$P_{n+1} = \frac{1}{P_n} + \frac{P_n}{2} - 1\,.$$

Calculate the two equilibrium points and determine the stability.

(11 marks)

Q7 Presently your weight is 76.66 kg. You consume x kg worth of calories each week. Assume your body burns off the equaivalent of 3% of its weight each week through normal metabolism. In addition, you burn off $1/4$ kg of weight through daily exercise each week.

(a) Show that your weight after n weeks can be written as

$$W_n = 0.97^n W_0 + (x - 0.25)\left(\frac{1 - 0.97^n}{1 - 0.97}\right),$$

with W_n is the weight after n weeks, and W_0 is the initial weight.

(9 marks)

(b) Compute x to two decimal places if you want your weight between 65.32 kg and 66.22 kg in a year (52 weeks).

(4 marks)

- END OF QUESTIONS -