UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2023/2024

COURSE NAME	:	VECTOR CALCULUS
COURSE CODE	:	BWA 20803
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	JULY 2024
DURATION	:	3 HOURS
INSTRUCTIONS	:	1. ANSWER ALL QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA ☐ Open book ☒ Closed book 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **FIVE (5)** PAGES

ANSWER ALL QUESTIONS

Q1 Sketch the surface area of the solid region bounded by the cone $z = \sqrt{x^2 + y^2} - 4$ and the plane $z = 4$.

(3 marks)

Q2 Use the Divergence theorem to evaluate

$$\iint_S \hat{F} \cdot \hat{n}\, dS,$$

where $\hat{F} = \langle x^3, y^3, 6z \rangle$ and S is the entire surface of the solid region bounded by the cone $z = \sqrt{x^2 + y^2} - 4$ and the plane $z = 4$. Take $\hat{n}$ to be the outward unit normal.

(8 marks)

Q3 Verify the Stoke's theorem

$$\iint_S (\hat{\nabla} \times \hat{F}) \cdot \hat{n}\, dS = \int_C \hat{F} \cdot d\tilde{r}$$

for the vector field $\hat{F} = \langle 4y, x^2, z^3 \rangle$ and C is the perimeter of a closed triangle of the plane $6x + 3y + z = 9$ in the first octant and counterclockwise when viewed from the positive z-axis. Take $\hat{n}$ to be the outward unit normal.

(14 marks)

Q4 $\hat{F} = \langle z + ye^x, 2ye^x, x \rangle$ is a vector field.

(a) Show that $\hat{F}$ obeys $\hat{\nabla} \times \hat{F} = \hat{0}$.

(2 marks)

(b) Then, find a corresponding scalar potential function ϕ, such that $\hat{F} = \nabla\phi$.

(5 marks)

(c) What can be concluded about the line integral

$$\int_A^B \hat{F} \cdot d\tilde{r}\,?$$

(1 mark)

(d) Calculate the work done in moving an object in this field from $A(0, -1, 4)$ to $B(3, 2, 1)$.

(2 marks)

Q5 Evaluate

$$\int_C [y - \cos(x)]dx + \sin(x)\, dy,$$

where C is the perimeter of the triangle formed by the lines $y = 1, x = 2$ and $y = 2x^2$ using Green's theorem. Sketch the triangle formed by the lines in your answers.

(8 marks)

Q6 Compute

$$\int_C \frac{y}{2}\, dx - xy\, dy$$

where C is the line segment from (3, 4) to (2, 1).

(4 marks)

Q7 Suppose that a particle travels along a circular helix in 3-space so that its position vector $\hat{r}(t) = \langle 2\cos \pi t, 2\sin \pi t, t\rangle$. Calculate the displacement and distance travelled by the particle during the time interval $2 \leq t \leq 5$.

(6 marks)

Q8 An ant moves around a vase in such a way that its position vector at time t is $\hat{r}(t) = \langle 2\cos t, 2\sin t\rangle$. Show that at each instant the acceleration vector is perpendicular to the velocity vector.

(3 marks)

Q9 Suppose that a particle moves through 3-dimensional space along the curve $\hat{r}(t) = \langle 2t, -2t^3, \cos 4t\rangle$ and that it is subjected to a force of $\hat{F} = \langle x^2, -4y, yz\rangle$ when it is at the point (x, y, z). Find $\hat{F}$ in terms of t for points on the path.

(3 marks)

Q10 Given an equation $\phi(x, y, z) = 3x^3 - y^2 \ln z$. Calculate $\nabla \cdot \nabla\phi$ at point (1,2,1).

(4 marks)

Q11 Find a unit normal vector to the surface $yx^2 - 2xz = 4$ at the point (2, -2, 3).

(4 marks)

Q12 If $\hat{B} = \langle z \ln x, -2x^2 \cos 2y, 2zx\rangle$, compute curl $\hat{B}$.

(3 marks)

Q13 If $\hat{r}(t) = \langle \sin 3t\,, \ln 2t + e^{2t}, 6t^4 \rangle$, calculate the vector $\hat{r}'(t_0)$ when $t_0 = \dfrac{\pi}{2}$.

(2 marks)

Q14 The equation of a curve is given by $x = a\cos t\,, y = a\sin t\,, z = t$. Find

(a) the curvature κ.

(6 marks)

(b) the radius of curvature ρ.

(2 marks)

- END OF QUESTIONS -

APPENDIX A

Formula:

$$\boldsymbol{T}(t) = \frac{d\boldsymbol{r}/dt}{\|d\boldsymbol{r}/dt\|} \qquad \boldsymbol{N} = \frac{d\boldsymbol{T}/dt}{\|d\boldsymbol{T}/dt\|}$$

$$\kappa = \frac{\|d\boldsymbol{T}/dt\|}{\|d\boldsymbol{r}/dt\|} \ \ or \ \ \kappa = \frac{\|\boldsymbol{r}'(t) \times \boldsymbol{r}''(t)\|}{\|\boldsymbol{r}'(t)\|^3}$$

$$\rho = \frac{1}{\kappa}$$

$$\tau = \frac{\|d\mathbf{B}/dt\|}{\|d\boldsymbol{r}/dt\|}$$

$$\sigma = \frac{1}{\tau}$$

$$B = \mathbf{T} \times \mathbf{N}$$

$$\int_C \xi(x,y,z)\,ds = \int_a^b \xi(x,y,z)\sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}\,dt$$

s denotes the arc length,

$$s = \int_C ds = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}\,dt$$

$$\int_C P(x,y,z)\,dx = \int_a^b P\big(x(t), y(t), z(t)\big)\,x'(t)\,dt$$

$$\int_C Q(x,y,z)\,dx = \int_a^b Q\big(x(t), y(t), z(t)\big)\,y'(t)\,dt$$

$$\int_C R(x,y,z)\,dx = \int_a^b R\big(x(t), y(t), z(t)\big)\,z'(t)\,dt$$

$$\int_C \hat{F} \cdot d\hat{r} = \iint_s \left(\nabla \times \hat{F}\right) \cdot \hat{n}\,dS$$

where $d\hat{r} = \langle dx, dy, dz \rangle$ and $\hat{F}(x,y,z) = \langle P(x,y,z), P(x,y,z), P(x,y,z) \rangle$

$$\int_C P(x,y)dx + Q(x,y)dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

$\nabla\phi_1 \cdot \nabla\phi_2 = \|\nabla\phi_1\|\|\nabla\phi_2\|\cos\theta$ where ϕ_1, ϕ_2 is differentiable vector functions of x, y and z.