Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2023/2024

COURSE NAME	:	NUMERICAL METHODS FOR FLUID DYNAMICS
COURSE CODE	:	BWA 33203
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	JULY 2024
DURATION	:	3 HOURS

1. ANSWER ALL QUESTIONS
2. THIS FINAL EXAMINATION IS CONDUCTED VIA
   ☐ Open book
   ☒ Closed book
3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **THREE (3)** PAGES

Q1 (a) Based on the ideas of:

- *(new value) = (old value) + (slope × step size),*
- *the slope is given by the first derivative,*
- *find the slope at the midpoint of the step size,*
- *update the new predictor value using the new slope,*

for differential equation $\frac{dy}{dx} = f(x, y)$.

Write the formula of 4th order Runge-Kutta method for:

(i) $\frac{dy_1}{dx} = f_1(x, y_1, y_2)$, $\frac{dy_2}{dx} = f_2(x, y_1, y_2)$.

(11 marks)

(ii) $\frac{dy_1}{dx} = f_1(x, y_1, y_2, y_3)$, $\frac{dy_2}{dx} = f_2(x, y_1, y_2, y_3)$, $\frac{dy_3}{dx} = f_3(x, y_1, y_2, y_3)$.

(8 marks)

(b) Hence, solve the following set of differential equations using 4th order Runge-Kutta by assuming that $y_1(1) = 4$ and $y_2(1) = 6$. Integrate to $x = 1.2$ with $\Delta x = 0.2$ (use 4 decimal places).

$$\frac{dy_1}{dx} = y_1, \quad \text{and} \quad \frac{dy_2}{dx} = 4 - y_2 + y_1.$$

(11 marks)

Q2 Consider

$$I = \int_{1.5}^{3.5} \int_{2}^{3} (x + y^2) dy\, dx.$$

Calculate I using:

(a) trapezoidal rule with $\Delta x = \Delta y = 0.5$.

(10 marks)

(b) 2-point Gauss quadrature, given that $\int_{-1}^{1} f(x)dx \approx f\left(-\frac{1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)$.

(10 marks)

Q3 Given the Simpson 1/3 for function $g(x)$ as follows:

$$\int_a^b g(x)dx = \frac{\Delta x}{3}\left[g(a) + 4\sum_{\substack{i=1 \\ i \text{ odd}}}^{n-1} g(a+i\Delta x) + 2\sum_{\substack{i=1 \\ i \text{ even}}}^{n-2} g(a+i\Delta x) + g(b) \right].$$

Evaluate $\int_0^1 \int_x^{2x} (x+y)dydx$ with $n = m = 4$ using Simpson 1/3.

(15 marks)

Q4 $T(x, y, t)$ is the temperature of a heated plate in the form of:

$$\frac{d^2T}{dx^2} + \frac{d^2T}{dy^2} = \frac{dT}{dt},$$

subject to boundary conditions as follows:

$t = 0$: $T(x, y, 0) = 0$ at $0 \le x \le 1$, and $0 \le y \le 1$,

$t > 0$:

$T(x, 0, t) = 0$, $T(x, 1, t) = 100$ at $0 \le x \le 1$,

$T(0, y, t) = 80$, $T(1, y, t) = 60$ at $0 \le y \le 1$.

Form the matrix obtained from the Crank-Nicolson method.

(15 marks)

- END OF QUESTIONS -