# UNIVERSITI TUN HUSSEIN ONN MALAYSIA

## FINAL EXAMINATION
## SEMESTER II
## SESSION 2023/2024

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| COURSE NAME | : | CALCULUS OF VARIATION AND OPTIMAL CONTROL |
| COURSE CODE | : | BWA 32903 |
| PROGRAMME CODE | : | BWA |
| EXAMINATION DATE | : | JULY 2024 |
| DURATION | : | 3 HOURS |
| INSTRUCTION | : | 1. ANSWER ALL QUESTIONS<br>2. THIS FINAL EXAMINATION IS CONDUCTED VIA<br>☐ Open book<br>☒ Closed book<br>3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK |

THIS QUESTION PAPER CONSISTS OF **THREE (3)** PAGES

**Q1** Consider a nonlinear control system,

$$\frac{dx}{dt} = f(x,u,t),$$

where $x$ and $u$ are scalar state and control variables, respectively, and the initial condition is $x(t_0) = x_0$. The performance index

$$J(t_0) = \phi(x(T),T) + \int_{t_0}^{T} L(x,u,t)\ dt$$

is to be minimized when an admissible control variable $u$ is determined. Here, $\phi$ and $L$ are the terminal and operating costs, respectively, and $[t_0, T]$ is the time interval.

(a) Describe the optimal control problem given above. You shall write only one sentence for your answer.

(4 marks)

(b) Derive the necessary conditions for the problem. You shall start from the augmented performance index.

(14 marks)

(c) Give one example of the optimal control problem.

(2 marks)

**Q2** Consider a nonlinear dynamical system

$$\frac{dx}{dt} = x + 2u,$$

with the initial value $x(0) = 1$. The performance index is given by

$$J = \int_0^1 (x^2 + u^2)\ dt.$$

(a) Define the Hamiltonian function.

(3 marks)

(b) Prove that the Hamilton-Jacobi-Bellman (HJB) equation is given by

$$0 = \frac{\partial J^*}{\partial t} + x^2 - \lambda^2 + \lambda x.$$

(12 marks)

(c) Generate the Riccati equation.

(5 marks)

**Q3** An optimal control problem has the plant dynamic given by

$$f(x,u,t)=\frac{x}{x+1}+u$$

and the weighting function is

$$L(x,u,t)=\frac{1}{2}u^2.$$

(a) Obtain the necessary conditions for the problem.

(10 marks)

(b) Determine the control function.

(2 marks)

(c) Verify that the state equation is $\dot{x}=(1+\lambda)(x+1)^{-2}$.

(8 marks)

**Q4** The length of a curve $x(t)$ is given by

$$J=\int_1^2 \sqrt{1+(\dot{x}(t))^2}\ dt,$$

where the boundary conditions are $x(1)=5$ and $x(2)=6$. This problem is a shortest distance between two points, aiming to find the curve $x(t)$ that minimizes the length $J$.

(a) Convert the problem into an optimal control problem.

(3 marks)

(b) Calculate the optimal control $u(t)$ for the problem.

(12 marks)

(c) Conclude that the curve $x(t)$ is a straight line.

(5 marks)

**– END OF QUESTIONS –**