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# UNIVERSITI TUN HUSSEIN ONN MALAYSIA

## FINAL EXAMINATION
## SEMESTER II
## SESSION 2023/2024

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|---|---|---|
| COURSE NAME | : | NUMERICAL METHOD |
| COURSE CODE | : | BFC25203 |
| PROGRAMME CODE | : | BFF |
| EXAMINATION DATE | : | JULY 2024 |
| DURATION | : | 3 HOURS |
| INSTRUCTIONS | : | 1. ANSWER ALL QUESTIONS<br>2. THIS FINAL EXAMINATION IS CONDUCTED VIA<br>☐ Open book<br>☒ Closed book<br>3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK |

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES



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**Q1** Solve the following:

(a) A speedometer reading of a battery-operated miniature sports car model is given as a function of time in **Table Q1.1.**

**Table Q1.1.** Reading value of a speedometer.

| Time (s) | 0 | 0.25 | 0.75 | 1.0 |
|---|---|---|---|---|
| Acceleration ($m/s^{-1}$) | 1 | 0.7788 | 0.4724 | 0.3679 |

(i) Approximate the acceleration at time 0.4s using Newton's divided-difference method.

(5 marks)

(ii) If data of acceleration, 0.607 $m/s^{-1}$ at a time of 0.5s is added to the table, approximate the new acceleration at a time of 0.4s using Newton's divided-difference method.

(10 marks)

(b) The current, $i$ of a circuit at a time, t is given in **Table Q1.2.** Given that the voltage drop across an inductor is, $V_L = L\frac{di}{dt}$ where inductance, L = 2, evaluate the voltage drop across the inductor at t= 5 seconds by using the appropriate difference formula.

**Table Q1.2.** Current, *i* of a circuit over the time, *t*.

| Time, *t* (seconds) | 3 | 5 | 7 | 9 | 11 |
|---|---|---|---|---|---|
| Current, *i* (A) | 4.600 | 8.030 | 11.966 | 16.885 | 19.904 |

(10 marks)

**Q2** Solve the following:

(a) State the differences Simpson's 1/3 and Simpson's 3/8 rule in numerical integration.

(2 marks)

(b) A stream cross section in shown is **Figure Q2.1.** The depth of stream (in meters) along the cross-section is measured by using digital gauge. Use the 10-segment Simpson 1/3 rule to determine the cross sectional area of the stream.

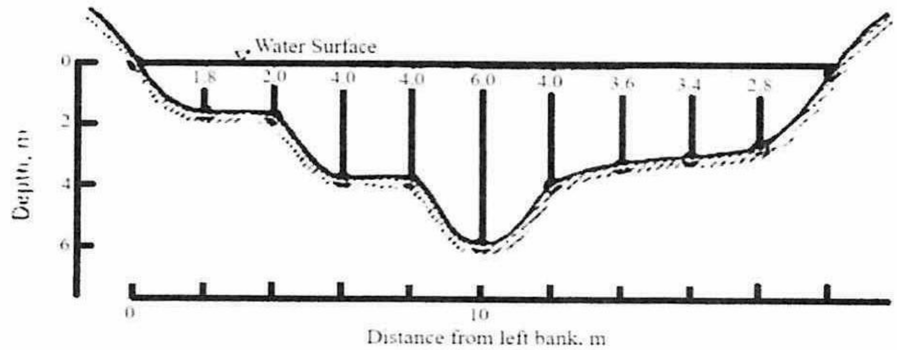


**Figure Q2.1.** Cross section of a stream.

(8 marks)

(c) Given $\int_a^b f(t)\,dt = \frac{b-a}{2}\int_{-1}^{1} g(x)dx$

(i) By letting $t = \frac{(b-a)x+(b+a)}{2}$, demonstrate that the integrals above are equivalent with the limit of integral from *a* to *b* to *-1* to *1*.

(9 marks)

(ii) The acceleration of a car at time t minutes is given by $\frac{\cos t}{t+1}$, determine the velocity travelled from t=1 to t=2 by using the 2-point and 3-point Gauss Quadrature formulas.

(6 marks)

**Q3** The power method and the finite difference method can be used in various engineering computational problems.

(a) The stability of the bridge construction can be determined computationally. Examine the natural frequency of a bridge system (dominant eigenvalue) and its corresponding eigenvector if the matrix form of bridge stability is below:

$$\begin{bmatrix} 6 & -2 & 1 \\ -2 & 5 & 0 \\ 2 & -2 & 3 \end{bmatrix}$$

Start with $v^{(0)} = (0 \quad 1 \quad 0)^T$ and stop the iteration until $|m_{k+1} - m_k| < 0.005$. Perform all calculations rounded to 3 decimal places.

(10 marks)

(b) Given the boundary-value problem $y'' - y' = 12x^2$, for $1 \leq x \leq 2$ with boundary conditions, $y(1) = 2$ and $y(2) = 17$. Take h = 0.2, sketch the diagram and derive the system of linear equation in matrix-vector form using finite-difference method without solving the whole matrix system.

(15 marks)

**Q4** In civil engineering, ordinary differential equations (ODEs) and partial differential equations (PDEs) are fundamentally encountered in various applications, such as structural analysis, fluid mechanics, heat transfer, and soil mechanics. Numerical methods play a crucial role in solving these equations to obtain approximate solutions for practical engineering problems.

(a) Consider a building subjected to ground motion during an earthquake. The equation of motion for the dynamic response of a single column structure is given by:

$$m\frac{du}{dt} + u(k + c) = F(t)$$

where the mass of the column, *m*, is 680kg; *u* is the displacement of the column in mm, *k* is stiffness of the column which is 4000 N/m. Assume that the damping coefficient of column, *c* is 100 Ns/m and the dynamic force applied to column due to ground motion, *F* is 8000 N.

(i) Estimate the displacement of the column until time $t$=3 seconds with $\Delta t$=1 and an initial displacement $u_0 = 0$ by using Runge-Kutta method.

(10 marks)

(ii) Evaluate whether the structure remains within a safe condition after 3 seconds if the maximum displacement limit for the column due to earthquakes is 0.5 meters. Justify your answer.

(2 marks)

(b) Given the heat equation use for assessing the heat gain of a concrete flat roof as follow:

$$\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 5, t > 0$$

where the boundary condition $u(0,t) = 25\ °C$ and $u(5,t) = 50 - e^t$ for $t > 0$. The initial condition $u(x,0) = 10x + \sin(x)$ for $0 \le x \le 5$. Using implicit Crank-Nicolson method, estimate the heat equations at the first level only ($t \le 5$) by taking $\Delta x = h = 1.0$ and $\Delta t = k = 1$. Perform all calculations rounded to 4 decimal places without solving the equations.

(13 marks)

**- END OF QUESTIONS -**

**APPENDIX A**

**Nonlinear equations**

Newton-Raphson Method : $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}, i = 0,1,2 ....$

**System of linear equations**

Gauss-Seidel Iteration:

$$x_i^{(k+1)} = \frac{b_i - \sum_{j=1}^{i-1} a_{ij}\ x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij}\ x_j^{(k)}}{a_{ii}}, \forall i = 1,2,3, \ldots, n.$$

**Interpolation**

**Lagrange Interpolating :** $L_i = \frac{(x-x_1)(x-x_2)}{(x_i-x_1)(x_i-x_2)} \cdots \frac{(x-x_n)}{(x_i-x_n)};$

$$f(x) = \sum_{i=1}^{n} L_i(x) f(x_i)$$

**Newton's divided difference**

$$f_i^{[0]} = f_i, \qquad i = 0,1, \ldots n$$

$$f_i^{[j]} = \frac{f_{i+1}^{[j-1]} - f_i^{[j-1]}}{x_{i+j} - x_i}, \qquad j = 1,2, \ldots n$$

$$f(x) \approx P_n(x)$$
$$= f_0^{[0]} + f_0^{[1]}(x - x_0) + f_0^{[2]}(x - x_0)(x - x_1)$$
$$+ \cdots f_0^{[n]}(x - x_0)(x - x_1) \ldots (x - x_{n-1})$$

**Natural Cubic Spline:**

$$\left.\begin{aligned} h_k &= x_{k+1} - x_k \\ d_k &= \frac{f_{k+1} - f_k}{h_k} \end{aligned}\right\}, k = 0,1,2,3, \ldots, n-1$$

$$b_k = 6(d_{k+1} - d_k), k = 0,1,2,3, .., n-2,$$

When; $m_0 = 0, m_n = 0,$

$$h_k m_k + 2(h_k + h_{k+1}) m_{k+1} + h_{k+1} m_{k+2} = b_k, k = 0,1,2,3, \ldots, n-2$$

$$S_k(x) = \frac{m_k}{6h_k}(x_{k+1} - x)^3 + \frac{m_{k+1}}{6h_k}(x - x_k)^3 + \left(\frac{f_k}{h_k} - \frac{m_k}{6}h_k\right)(x_{k+1} - x)$$

$$+ \left(\frac{f_{k+1}}{h_k} - \frac{m_{k+1}}{6}h_k\right)(x - x_k) \quad , \; k = 0,1,2,3, \ldots n-1$$

**Numerical Differentiation**

2-point forward difference: $f'(x) \approx \frac{f(x+h)-f(x)}{h}$

2-point backward difference: $f'(x) \approx \frac{f(x)-f(x-h)}{h}$

3-point central difference: $f'(x) \approx \frac{f(x+h)-f(x-h)}{2h}$

3-point forward difference: $f'(x) \approx \frac{-f(x+2h)+4f(x+h)-3f(x)}{2h}$

3-point backward difference: $f'(x) \approx \frac{3f(x)-4f(x-h)+f(x-2h)}{2h}$

5-point difference formula: $f'(x) \approx \frac{-f(x+2h)+8f(x+h)-8f(x-h)+f(x-2h)}{12h}$

3-point central difference: $f''(x) \approx \frac{f(x+h)-2f(x)+f(x-h)}{h^2}$

5-point difference formula: $f''(x) \approx \frac{-f(x+2h)+16f(x+h)-30f(x)+16f(x-h)-f(x-2h)}{12h^2}$

**Numerical Integration**

Simpson $\frac{1}{3}$ Rule : $\int_a^b f(x)dx \approx \frac{h}{3}\left[f_0 + f_n + 4\sum_{\substack{i=1 \\ i\,odd}}^{n-1} f_i + 2\sum_{\substack{i=2 \\ i\,even}}^{n-2} f_i\right]$

Simpson $\frac{3}{8}$ Rule : $\int_a^b f(x)dx \approx \frac{3}{8}h\,[(f_0 + f_n) + 3(f_1 + f_2 + f_4 + f_5 + \cdots + f_{n-2} + f_{n-1}) + 2(f_3 + f_6 + \cdots + f_{n-3})]$

2-point Gauss Quadrature: $\int_a^b g(x)dx = \left[g\left(-\frac{1}{\sqrt{3}}\right) + g\left(\frac{1}{\sqrt{3}}\right)\right]$

3-point Gauss Quadrature: $\int_a^b g(x)dx = \left[\frac{5}{9}g\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9}g(0) + \frac{5}{9}g\left(\sqrt{\frac{3}{5}}\right)\right]$

**Eigen Value**

Power Method : $v^{(k+1)} = \frac{1}{m_{k+1}}Av^{(k)}, k = 0,1,2 \ldots.$

Shifted Power Method : $v^{(k+1)} = \frac{1}{m_{k+1}}A_{shifted}v^{(k)}, k = 0,1,2 \ldots.$

**Ordinary Differential Equation**

Fourth-order Runge-Kutta Method : $y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

where $k_1 = hf(x_i, y_i)$ $k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2})$

$k_3 = hf(x_i + \frac{h}{2}, y_i + \frac{k_2}{2})$ $k_4 = hf(x_i + h, y_i + k_3)$

**Partial Differential Equation**

Heat Equation: Finite Difference Method

$$\left(\frac{\partial u}{\partial t}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \quad \frac{u_{i,j+1} - u_{i.j}}{k} = c^2 \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

Heat Equation: Crank-Nicolson Implicit Finite-Difference Method

$$\left(\frac{\partial u}{\partial t}\right)_{i,j+\frac{1}{2}} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j+\frac{1}{2}}$$

$$\frac{u_{i,j+1} - u_{i.j}}{k} = \frac{c^2}{2}\left(\frac{u_{i-1,j+1} - 2u_{i,j+1} + u_{i+1,j+1}}{h^2} + \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}\right)$$

Poisson Equation: Finite Difference Method

$$\left(\frac{\partial^2 u}{\partial x^2}\right)_{i,j} + \left(\frac{\partial^2 u}{\partial y^2}\right)_{i,j} = f_{i,j} \quad \frac{u_{i+1,j} - 2u_{i.j} + u_{i-1,j}}{h^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{k^2} = 0$$

Wave Equation: Finite Difference Method

$$\left(\frac{\partial^2 u}{\partial t^2}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \quad \frac{u_{i,j-1} - 2u_{i.j} + u_{i,j+1}}{k^2} = c^2 \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$