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Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2023/2024

COURSE NAME	:	CALCULUS
COURSE CODE	:	BFC 15003
PROGRAMME CODE	:	BFF
EXAMINATION DATE	:	JULY 2024
DURATION	:	3 HOURS
INSTRUCTIONS	:	1. ANSWER ALL QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA ☐ Open book ☒ Closed book 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

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Q1 Answer all the following questions.

(a) If $y = \ln(1-2x+2x^2)^{\frac{2}{3}}$, determine the $\frac{dy}{dx}$.

(5 marks)

(b) Find $\frac{dy}{dx}$ if $y = [x^3][\sin^3(x)] + 2\pi$.

(6 marks)

(c) Given parametric equations $x = \frac{1+t^2}{t}$ and $y = \frac{4t+6}{2t}$. By using Chain Rule, find the $\frac{dy}{dx}$.

(7 marks)

(d) By using the Implicit Differentiation, determine the $\frac{dy}{dx}$ for $\ln y + 2xy^2 = 100 - x$.

(7 marks)

Q2 Answer all the following questions.

(a) Find the intervals where the function $f(x) = \frac{1}{3}x^3 + \frac{1}{3}x^2 - 3x + 4$ is increasing and decreasing.

(5 marks)

(b) Determine the intervals where the function $f(x) = 4x^3 - 5x^2 - 10x + 11$ is concave upwards and downwards.

(5 marks)

(c) The radius, r cm of a spherical balloon at time t seconds is given by $r = 3 + \frac{1}{1+t}$.

(i) Identify the initial radius of the sphere.

(1 mark)

(ii) Determine the rate of change for the surface area of the sphere when $t = 2$.

(6 marks)

(d) Sand is falling at a constant rate of 8 m^3/min, forming a cone shape as it falls to the ground, where the height of the cone is twice its radius. Determine the rate of change of the cone's height when its height reaches 2 meters by using $V = \frac{1}{3}\pi r^2 h$.

(8 marks)

Q3 Evaluate the following integrals using a suitable integration method.

(a) $\int_1^8 \frac{(s^{2/3}+5)^3}{\sqrt[3]{s}}\,ds$.

(6 marks)

(b) $\int \frac{5}{2x^2+7x-4}\,dx$.

(6 marks)

(c) $\int x\cos(5x-2)\,dx$.

(6 marks)

(d) Show that $\int_0^{\pi} \frac{5}{\pi}\sec(5t-\pi)\tan(5t-\pi)\,dt = \frac{2}{\pi}$.

(7 marks)

Q4 Answer all the following questions.

(a) **Figure Q4.1** below shows the results from flexural testing of a concrete prism to investigate the fracture energy of foam concrete with density of 1600 kg/m[3]. Fracture energy, G_f can be calculated by finding the area under the graph up to the maximum load. The relationship between flexural load – deflection can be express by the function of $y = 15.429x^2 - 0.1942x + 0.119$.

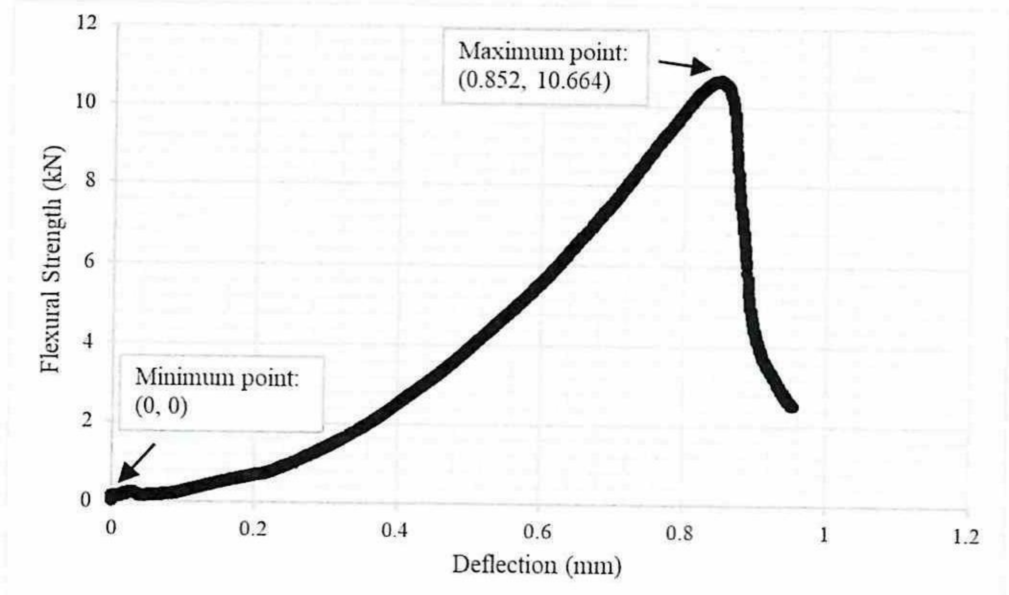

Figure Q4.1 Flexural Strength – Deflection of Foamed Concrete (1600 kg/m[3])

(i) Determine the fracture energy of the foam concrete (area under the graph).

(4 marks)

(ii) Fracture energy can also be calculated by using formula $G_f = \dfrac{U_o + mgd_o}{B(W - a_o)}$. From this formula, the value of fracture energy obtained was 3.325 kN/mm. Calculate the percentage different and justify your answer in **Q4(a)(i)** with the calculated G_f using the formula.

(2 marks)

(b) Find the area of the surface generated by revolving the curve $y = \sqrt{2x - x^2}$ for $0.5 \le x \le 1.5$, about the x - axis.

(8 marks)

(c) Calculate the radius of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{m}$ at the point of $\left(\dfrac{m}{2}, \dfrac{m}{2}\right)$.

(11 marks)

- END OF QUESTIONS -

APPENDIX A

Table APPENDIX A.1

Differentiation Rules	Indefinite Integrals
$\frac{d}{dx}[k]=0, \quad k$ constant	$\int k\,dx = kx + C$
$\frac{d}{dx}[x^n]=nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1}+C, \quad n \neq -1$
$\frac{d}{dx}[\ln\|x\|]=\frac{1}{x}$	$\int \frac{dx}{x} = \ln\|x\| + C$
$\frac{d}{dx}[\cos x]=-\sin x$	$\int \sin x\,dx = -\cos x + C$
$\frac{d}{dx}[\sin x]=\cos x$	$\int \cos x\,dx = \sin x + C$
$\frac{d}{dx}[\tan x]=\sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
$\frac{d}{dx}[\cot x]=-\mathrm{cosec}^2 x$	$\int \mathrm{cosec}^2 x\,dx = -\cot x + C$
$\frac{d}{dx}[\sec x]=\sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$
$\frac{d}{dx}[\mathrm{cosec}\, x]=-\mathrm{cosec}\, x \cot x$	$\int \mathrm{cosec} x \cot x\,dx = -\mathrm{cosec} x + C$
$\frac{d}{dx}[e^x]=e^x$	$\int e^x\,dx = e^x + C$

Table APPENDIX A.2

Trigonometric Identities
$\cos^2 x + \sin^2 x = 1$
$1 + \tan^2 x = \sec^2 x$
$\cot^2 x + 1 = \csc^2 x$
$\sin 2x = 2\sin x \cos x$
$\cos 2x = \cos^2 x - \sin^2 x$
$\cos 2x = 2\cos^2 x - 1$
$\cos 2x = 1 - 2\sin^2 x$
$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$
$\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$
$2\sin x \cos y = \sin(x+y) + \sin(x-y)$
$2\sin x \sin y = -\cos(x+y) + \cos(x-y)$
$2\cos x \cos y = \cos(x+y) + \cos(x-y)$
Logarithm
$a^x = e^{x \ln a}$
$\log_a x = \dfrac{\log_b x}{\log_b a}$

APPENDIX B

Area between two curves

Case 1- Integrating with respect to *x*: $A = \int_a^b [f(x) - g(x)]dx$

Case 2- Integrating with respect to *y*: $A = \int_c^d [f(y) - g(y)]dy$

Area of surface of revolution

Case 1- Revolving the portion of the curve about *x*-axis: $S = 2\pi \int_a^b y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

Case 2- Revolving the portion of the curve about *y*-axis: $S = 2\pi \int_c^d x\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

Parametric curve- Revolving the curve about *x*-axis: $S = 2\pi \int_a^b y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Parametric curve- Revolving the curve about *y*-axis: $S = 2\pi \int_c^d x\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Arc length

x-axis: $L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

y-axis: $L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

Parametric curve: $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Curvature

Curvature, $K = \dfrac{\left|\frac{d^2y}{dx^2}\right|}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}$

Radius of curvature, $\rho = \frac{1}{K}$

Curvature of parametric curve

Curvature, $K = \dfrac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{(\dot{x}^2 + \dot{y}^2)^{3/2}}$

Radius of curvature, $\rho = \frac{1}{K}$