UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2023/2024

COURSE NAME	:	CIVIL ENGINEERING STATISTICS
COURSE CODE	:	BFC 34303
PROGRAMME CODE	:	BFF
EXAMINATION DATE	:	JULY 2024
DURATION	:	3 HOURS
INSTRUCTIONS	:	1. ANSWER **ALL** QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA ☐ Open book ☒ Closed book 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK.

THIS QUESTION PAPER CONSISTS OF **FOURTEEN (14)** PAGES

Q1 A pavement engineer claims that rubber modified asphalt (RMA) can reduce traffic noise. To support the claim, a study on roads built using conventional asphalt and RMA was conducted. The results of the study are shown in **Table Q1.1**. Using a suitable statistical analysis, determine if the claim can be supported at the 0.05 significance level.

Table Q1.1 Tire-pavement noise levels recorded at various roads built using conventional and rubber modified asphalt

Tire-Pavement Noise Level (dB)	
Conventional Asphalt	**Rubber Modified Asphalt**
83.4	71.2
80.7	73.4
75.6	64.6
72.1	67.3
82.9	69.5
75.8	71.8
74.2	75.3
89.6	75.6
77.8	80.4
70.3	76.5

(25 marks)

Q2 The data obtained in a study on the number of absences and final examination score of ten randomly selected students from a statistics class. The data are presented in **Table Q2.1.**

Table Q2.1 Number of absences and final examination score

Students	Number of absences, X	Final Examination Score, Y
1	8	78
2	5	90
3	12	58
4	9	74
5	15	43
6	2	86
7	6	82
8	7	80
9	4	91
10	18	35

a) Draw a scatter plot. Provide the drawing in answer script.

(2 marks)

b) Develop a linear regression equation that relates the number of absences and final examination score. Interpret the result.

(10 marks)

c) Find the coefficient of correlation and coefficient of determination. Interpret the results.

(8 marks)

d) Predict the final test result for a student who has missed ten days of class this semester.

(1 marks)

e) Calculate the standard error of the estimate. Interpret the result.

(4 marks)

Q3 A study was conducted to evaluate moisture failure of asphaltic concrete (grade AC14) incorporating crumb rubber (CR). To examine whether different percentages of CR will distinguish the effect, three groups of samples were prepared by incorporating 2%, 3% and 4% CR, while the control group contained 0% CR. **Table Q3.1** shows the Indirect Tensile Strength results of the samples tested at standard temperature. At the 0.01 significance level, determine if there is a difference in the mean values for the four groups using one-way ANOVA test.

Table Q3:1 Indirect tensile strength of asphalt concrete incorporating different percentages of crumb rubber

	Crumb rubber content			
Sample	**0%**	**2%**	**3%**	**4%**
1	0.92	1.12	1.18	1.90
2	0.90	1.48	1.16	1.66
3	0.86	1.28	1.23	1.54
4	0.85	1.21	1.45	1.32
5	1.15	1.18	1.30	1.56
6	0.83	1.12	1.60	1.17

a) State the null hypothesis (H_o) and alternative hypothesis (H_a).

(1 mark)

b) Calculate Sum of squares total (SS).

(9 marks)

c) Determine Sum of squares treatment (SST).

(10 marks)

d) Determine the F-ratio.

(4 marks)

e) State the conclusion of this hypothesis test.

(1 mark)

Q4 (a) Describe **TWO (2)** differences between parametric and non-parametric tests.

(4 marks)

(b) **Table Q4.1** shows the number of illegal turnings made by drivers at the main entrance road of Lotus's Taman Universiti parking lot in the morning and afternoon. The manager of Lotus's is particularly interested in determining whether there are more illegal turnings made in the morning compared to in the afternoon. At the 0.05 significance level, can we conclude that there are more illegal turnings made in the morning than in the afternoon by using Mann Whitney Test?

Table Q4.1 Number of illegal turnings

Morning	Afternoon
15	1
3	6
9	11
6	12
12	8
2	2
11	6
8	7
10	13
4	9
5	

(21 marks)

- END OF QUESTIONS -

APPENDIX A

The following equations may be useful. The symbols have their usual meaning.

Mean and Variance of Ungrouped Data

$$\bar{x} = \frac{\sum x}{n} \qquad s^2 = \frac{\sum (x - \bar{x})^2}{n - 1}$$

Mean and Variance of Grouped Data

$$\bar{x} = \frac{\sum fx}{\sum f} \qquad s^2 = \frac{\sum fx^2 - \frac{(\sum fx)^2}{\sum f}}{(\sum f) - 1}$$

One-Sample Hypothesis Testing (z-test and t-test)

$$z = \frac{\bar{X} - \mu_o}{\sigma / \sqrt{n}} \qquad z = \frac{\bar{X} - \mu_o}{s / \sqrt{n}} \qquad t = \frac{\bar{X} - \mu_o}{s / \sqrt{n}}$$

Two-Sample Hypothesis Testing (z-test and t-test)

$$z = \frac{\bar{X}_X - \bar{X}_Y}{\sqrt{\frac{{s_X}^2}{n} + \frac{{s_Y}^2}{m}}} \qquad t = \frac{\bar{X}_X - \bar{X}_Y}{\sqrt{\frac{{s_P}^2}{n} + \frac{{s_P}^2}{m}}} \qquad {s_P}^2 = \frac{(n-1){s_X}^2 + (m-1){s_Y}^2}{n + m - 2}$$

Simple Linear Regression

$$Y = a + bX \qquad a = \frac{\sum Y}{n} - b\frac{\sum X}{n} \qquad e = Y - \hat{Y}$$

$$b = \frac{n(\sum XY) - (\sum X)(\sum Y)}{n(\sum X^2) - (\sum X)^2} \qquad r = \frac{n(\sum XY) - (\sum X)(\sum Y)}{\sqrt{[n(\sum X^2) - (\sum X)^2][n(\sum Y^2) - (\sum Y)^2]}}$$

$$s_{y.x} = \sqrt{\frac{\sum Y^2 - a(\sum Y) - b(\sum XY)}{n - 2}}$$

F-test

$$F = \frac{{s_1}^2}{{s_2}^2} \qquad v_1 = n_1 - 1 \qquad v_2 = n_2 - 1$$

One Way ANOVA Test

$$SS = \sum X^2 - \frac{(\sum X)^2}{n} \qquad SST = \sum\left(\frac{{T_c}^2}{n_c}\right) - \frac{(\sum X)^2}{n} \qquad SSE = SS - SST$$

$$MST = \frac{SST}{k-1} \qquad MSE = \frac{SSE}{n-k} \qquad F = \frac{MST}{MSE}$$

$$v_1 = k - 1 \qquad v_2 = n - k$$

Chi-Square Test

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} \qquad df = n - 1$$

Chi-Square Goodness of Fit Test

$$\chi^2 = \sum\left[\frac{(f_o - f_e)^2}{f_e}\right] \qquad df = k - 1$$

Chi-Square Contingency Table Analysis

$$\chi^2 = \sum\left[\frac{(f_o - f_e)^2}{f_e}\right] \qquad df = (r-1)(c-1)$$

Mann-Whitney Test / Mann-Whitney U Test

$$z = \frac{W - \frac{n_1(n_1+n_2+1)}{2}}{\sqrt{\frac{n_1 n_2(n_1+n_2+1)}{12}}} \qquad U_1 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R_1 \qquad U_2 = n_1 n_2 + \frac{n_2(n_2+1)}{2} - R_2$$

APPENDIX B

Table APPENDIX B.1 Standard Normal Distribution (Right-Tail) showing P (Z > z)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002

Table APPENDIX B.2 Standard Normal Distribution showing P (0 < Z < z)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998

Table APPENDIX B.3 Critical Values of the Student's t distribution

df	Level of significance for One-Tailed Test, α						
	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
	Level of significance for Two-Tailed Test, α						
	0.2	0.1	0.05	0.02	0.01	0.002	0.001
1	3.078	6.314	12.076	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646

Table APPENDIX B.4 Critical Values of the *F* distribution

	$\alpha = 0.05$									
v_2	v_1									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24

Table APPENDIX B.5 Critical Values of the Chi-square, χ^2 distribution

df	α					
	0.1	**0.05**	**0.025**	**0.01**	**0.005**	**0.001**
1	2.706	3.841	5.024	6.635	7.879	10.828
2	4.605	5.991	7.378	9.210	10.597	13.816
3	6.251	7.815	9.348	11.345	12.838	16.266
4	7.779	9.488	11.143	13.277	14.860	18.467
5	9.236	11.070	12.833	15.086	16.750	20.515
6	10.645	12.592	14.449	16.812	18.548	22.458
7	12.017	14.067	16.013	18.475	20.278	24.322
8	13.362	15.507	17.535	20.090	21.955	26.124
9	14.684	16.919	19.023	21.666	23.589	27.877
10	15.987	18.307	20.483	23.209	25.188	29.588
11	17.275	19.675	21.920	24.725	26.757	31.264
12	18.549	21.026	23.337	26.217	28.300	32.909
13	19.812	22.362	24.736	27.688	29.819	34.528
14	21.064	23.685	26.119	29.141	31.319	36.123
15	22.307	24.996	27.488	30.578	32.801	37.697
16	23.542	26.296	28.845	32.000	34.267	39.252
17	24.769	27.587	30.191	33.409	35.718	40.790
18	25.989	28.869	31.526	34.805	37.156	42.312
19	27.204	30.144	32.852	36.191	38.582	43.820
20	28.412	31.410	34.170	37.566	39.997	45.315
21	29.615	32.671	35.479	38.932	41.401	46.797
22	30.813	33.924	36.781	40.289	42.796	48.268
23	32.007	35.172	38.076	41.638	44.181	49.728
24	33.196	36.415	39.364	42.980	45.559	51.179
25	34.382	37.652	40.646	44.314	46.928	52.620
26	35.563	38.885	41.923	45.642	48.290	54.052
27	36.741	40.113	43.195	46.963	49.645	55.476
28	37.916	41.337	44.461	48.278	50.993	56.892
29	39.087	42.557	45.722	49.588	52.336	58.301
30	40.256	43.773	46.979	50.892	53.672	59.703

Table APPENDIX B.6 Critical Values of the Mann-Whitney U (Two-tailed)

		n_1												
n_2	α	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
3	**0.05**	0	0	0	1	1	2	2	3	3	4	4	5	5
	0.01	0	0	0	0	0	0	0	0	0	1	1	1	2
4	**0.05**	0	0	1	2	3	4	4	5	6	7	8	9	10
	0.01	0	0	0	0	0	1	1	2	2	3	3	4	5
5	**0.05**	0	1	2	3	5	6	7	8	9	11	12	13	14
	0.01	0	0	0	1	1	2	3	4	5	6	7	7	8
6	**0.05**	1	2	3	5	6	8	10	11	13	14	16	17	19
	0.01	0	0	1	2	3	4	5	6	7	9	10	11	12
7	**0.05**	1	3	5	6	8	10	12	14	16	18	20	22	24
	0.01	0	0	1	3	4	6	7	9	10	12	13	15	16
8	**0.05**	2	4	6	8	10	13	15	17	19	22	24	26	29
	0.01	0	1	2	4	6	7	9	11	13	15	17	18	20
9	**0.05**	2	4	7	10	12	15	17	20	23	26	28	31	34
	0.01	0	1	3	5	7	9	11	13	16	18	20	22	24
10	**0.05**	3	5	8	11	14	17	20	23	26	29	33	36	39
	0.01	0	2	4	6	9	11	13	16	18	21	24	26	29
11	**0.05**	3	6	9	13	16	19	23	26	30	33	37	40	44
	0.01	0	2	5	7	10	13	16	18	21	24	27	30	33
12	**0.05**	4	7	11	14	18	22	26	29	33	37	41	45	49
	0.01	1	3	6	9	12	15	18	21	24	27	31	24	37
13	**0.05**	4	8	12	16	20	24	28	33	37	41	45	50	54
	0.01	1	3	7	10	13	17	20	24	27	31	34	38	42
14	**0.05**	5	9	13	17	22	26	31	36	40	45	50	55	59
	0.01	1	4	7	11	15	18	22	26	30	34	38	42	46
15	**0.05**	5	10	14	19	24	29	34	39	44	49	54	59	64
	0.01	2	5	8	12	16	20	24	29	33	37	42	46	51

Table APPENDIX B.7 Critical Values of the Mann-Whitney U (One-tailed)

		n_1												
n_2	α	3	4	5	6	7	8	9	10	11	12	13	14	15
3	0.05	0	0	1	2	2	3	4	4	5	5	6	7	7
	0.01	0	0	0	0	0	0	1	1	1	2	2	2	3
4	0.05	0	1	2	3	4	5	6	7	8	9	10	11	12
	0.01	0	0	0	1	1	2	3	3	4	5	5	6	7
5	0.05	1	2	4	5	6	8	9	11	12	13	15	16	18
	0.01	0	0	1	2	3	4	5	6	7	8	9	10	11
6	0.05	2	3	5	7	8	10	12	14	16	17	18	21	23
	0.01	0	1	2	3	4	6	7	8	9	11	12	13	15
7	0.05	2	4	6	8	11	13	15	17	19	21	24	26	28
	0.01	0	1	3	4	6	7	9	11	12	14	16	17	19
8	0.05	3	5	8	10	13	15	18	20	23	26	28	31	33
	0.01	0	2	4	6	7	9	11	13	15	17	20	22	24
9	0.05	4	6	9	12	15	18	21	24	27	30	33	36	39
	0.01	1	3	5	7	9	11	14	16	18	21	23	26	28
10	0.05	4	7	11	14	17	20	24	27	31	34	37	41	44
	0.01	1	3	6	8	11	13	16	19	22	24	27	30	33
11	0.05	5	8	12	16	19	23	27	31	34	38	42	46	50
	0.01	1	4	7	9	12	15	18	22	25	28	31	34	37
12	0.05	5	9	13	17	21	26	30	34	38	42	47	51	55
	0.01	2	5	8	11	14	17	21	24	28	31	35	38	42
13	0.05	6	10	15	19	24	28	33	37	42	47	51	56	61
	0.01	2	5	9	12	16	20	23	27	31	35	39	43	47
14	0.05	7	11	16	21	26	31	36	41	46	51	56	61	66
	0.01	2	6	10	13	17	22	26	30	34	38	43	47	51
15	0.05	7	12	18	23	28	33	39	44	50	55	61	66	72
	0.01	3	7	11	15	19	24	28	33	37	42	47	51	56