Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2023/2024

COURSE NAME	:	MATHEMATICS FOR ENGINEERING TECHNOLOGY I
COURSE CODE	:	BDJ 12203
PROGRAMME CODE	:	BDJ
EXAMINATION DATE	:	JULY 2024
DURATION	:	3 HOURS
INSTRUCTIONS	:	1. ANSWER ALL QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA ☐ Open book ☒ Closed book 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

Q1 Limits is a fundamental concepts behind derivatives and calculus in general, which is used to describe the behavior of $f(x)$ as x approaches a.

(a) Find the following limits.

(i) $\lim_{x \to h} \dfrac{x^2 - 2hx + h^2}{x - h}$.

(4 marks)

(ii) $\lim_{x \to 0} \dfrac{6}{x}\left(\dfrac{1}{4+x} - \dfrac{1}{4-x}\right)$.

(4 marks)

(iii) $\lim_{x \to \infty} \sqrt{x^8 + 6x^4} - x^4$.

(6 marks)

(b) Compute the values of A and B so that $g(x)$ is continuous everywhere.

$$g(x) = \begin{cases} \sin\left(\dfrac{\pi}{2x}\right) + Ax & , \quad x < 1, \\ x^2 - Bx + 2 & , \quad x = 1, \\ \dfrac{1}{2x-3} & , \quad x > 1. \end{cases}$$

(6 marks)

Q2 Derivatives is very important since it is widely used in science and engineering technology.

(a) Find $\frac{dy}{dx}$ for $y = \tan\left(\frac{x}{x-1}\right)$ at $x = 0$.

(5 marks)

(b) Differentiate $y = \frac{3x}{\sqrt{(x+1)(x+2)}}$, using logarithmic differentiation.

(7 marks)

(c) If $y = e^x \sin 3x$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. Hence show that

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 10y = 0.$$

(8 marks)

Q3 The antiderivative or integration has also been a major parts of the key concepts in mathematics for engineering technology.

(a) Solve $\int \frac{x+4}{(2-x)^2}\,dx$.

(6 marks)

(b) Compute $\int_0^{\frac{\pi}{4}} x\sec^2 x\ dx$ using the tabular method.

(6 marks)

(c) Find the integrals $\int \frac{dx}{3+5\,\mathrm{si}}$ using substitution $t = \tan\frac{1}{2}x$.

(8 marks)

Q4 The linear approximation basically are related to power series expansion.

(a) Given a function $f(x) = \cos^2 x$.

(i) Determine the first four nonzero term of Maclaurin series for $f(x)$.

(8 marks)

(ii) Hence, referring your answer in **Q4(a)(i)**, compute $\frac{d}{dx}(x^5\cos^2 x)$.

(4 marks)

(iii) Also, referring your answer in **Q4(a)(i)**, evaluate $\int_0^1 \cos^2 x\ dx$.

(3 marks)

(b) Use the ratio test to determine whether the given series are converge or not.

$$\sum_{n=1}^{\infty} \frac{5^5}{4^{3n+1}(n+1)}$$

(5 marks)

Q5 Given a vector-valued function $\mathrm{r}(t) = t\mathbf{i} + \ln\sin 2t\,\mathbf{j} + \cos 2t\,\mathbf{k}$.

(a) Find the limits of $\mathrm{r}(t)$ as t approaching $\pi/4$.

(4 marks)

(b) Calculate $\mathbf{r}'(t)$ at $t = \pi/4$.

(6 marks)

(c) Find the vector equation of the tangent line of $\mathrm{r}(t)$ at $t = \pi/4$.

(5 marks)

(d) Consider a vector-valued function $\mathrm{s}(t) = 2\mathbf{i} + \ln \sin 2t\,\mathbf{j}$. Hence, solve $\int \mathrm{r}(t) - \mathrm{s}(t)\ dt$.

(5 marks)

- END OF QUESTIONS -

APPENDIX A

INDEFINITE INTEGRALS

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \csc^2 x \, dx = -\cot x + C$$

$$\int \sec x \tan x \, dx = \sec x + C$$

$$\int \csc x \cot x \, dx = -\csc x + C$$

$$\int e^x dx = e^x + C$$

$$\int \cosh x \, dx = \sinh x + C$$

$$\int \sinh x \, dx = \cosh x + C$$

$$\int \text{sech}^2 x \, dx = \tanh x + C$$

$$\int \text{csch}^2 x \, dx = -\coth x + C$$

$$\int \text{sech}\, x \tanh x \, dx = -\text{sech}\, x + C$$

$$\int \text{csch}\, x \coth x \, dx = -\text{csch}\, x + C$$

TRIGONOMETRY SUBSTITUTION

Expression	Trigonometry
$\sqrt{x^2 + k^2}$	$x = k \tan \theta$
$\sqrt{x^2 - k^2}$	$x = k \sec \theta$
$\sqrt{k^2 - x^2}$	$x = k \sin \theta$

TRIGONOMETRY SUBSTITUTION

$t = \tan\frac{1}{2}x$

$\sin x = \dfrac{2t}{1+t^2}$	$\cos x = \dfrac{1-t^2}{1+t^2}$
$\tan x = \dfrac{2t}{1-t^2}$	$dx = \dfrac{2dt}{1+t^2}$

TRIGONOMETRIC IDENTITIES

$$\cos^2 x + \sin^2 x = 1$$

$$\sin 2x = 2\sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 2\cos^2 x - 1$$

$$= 1 - 2\sin^2 x$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

$$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$2\sin ax \cos bx = \sin(a+b)x + \sin(a-b)x$$

$$2\sin ax \sin bx = \cos(a-b)x - \cos(a+b)x$$