UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2023/2024

COURSE NAME	:	STATISTICS FOR ENGINEERING TECHNOLOGY
COURSE CODE	:	BDJ 22502
PROGRAMME CODE	:	BDJ
EXAMINATION DATE	:	JULY 2024
DURATION	:	2 HOURS AND 30 MINUTES
INSTRUCTIONS	:	1. ANSWER ALL QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA ☐ Open book ☒ Closed book 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

Q1 A probability distribution consists of the values a random variable can assume and the corresponding probabilities of the values, which are determine theoretically or by observation.

(a) Let h be a constant and consider the probability distribution function.

$$f(x) = \begin{cases} h(x - x^2) & , \quad 0 \leq x \leq 1, \\ 0 & , \quad \text{otherwise.} \end{cases}$$

Find

(i) the value of h,

(4 marks)

(ii) expectation of X, $E(X)$,

(4 marks)

(iii) variance of X, $Var(X)$,

(5 marks)

(iv) $E(2X + 5)$,

(4 marks)

(v) $Var(2X - 5)$.

(3 marks)

(b) In an accelerator center, an experiment needs a 1.41 cm thick aluminum cylinder. Suppose that the thickness of a cylinder has a normal distribution with a mean 1.41 cm and a standard deviation of 0.01 cm. What is the probability that a thickness of a cylinder is greater than 1.42 cm?

(5 marks)

Q2 Probability distributions are very important since it is widely used in science and engineering technology.

(a) The comprehensive strength of samples of cement can be modeled by a normal distribution with a mean of 6000 kilograms per square centimeter and a standard deviation of 100 kilograms per square centimeter.

(i) What is the probability that a sample strength is less than 6250 kg/cm^2?

(4 marks)

(ii) If 10% of the cements are considered stronger, what is the minimum comprehensive strength of the cement?

(4 marks)

(b) A random sample of size sixteen is selected from a normal population with a mean of 75 and a standard deviation of eight from sample A. A second sample of size nine is selected from another normal population with a mean of 70 and a standard deviation of twelve from sample B. Let $\bar{X}_A$ and $\bar{X}_B$ be the two sample means.

(i) Compute the probability that the mean difference between sample A and sample B will be exceed four.

(6 marks)

(ii) Compute the probability that the mean difference between sample A and sample B will be between 3.5 and 5.5.

(6 marks)

(c) The average number of traffic accidents on a certain section of highway is two per week.

(i) State the variance for this distribution.

(1 marks)

(ii) Compute the probability of at most one accident on this section of highway during a 14 days period.

(4 marks)

Q3 Hypothesis testing has also been a major part of statistics for engineering technology.

(a) Define the critical region and critical values.

(4 marks)

(b) Assume that we are conducting a hypothesis test of the claim that $\mu < 0.10$. Here are the null and alternative hypotheses, $H_0: \mu = 0.10$ and $H_0: \mu < 0.10$. Give the statements that identifies:

(i) Type I error,

(2 marks)

(ii) Type II error.

(2 marks)

(c) Suppose a statistics instructor believes that there is no significant difference between the average class scores of her two classes on Exam 2. The average and standard deviation for her Class A of 35 students were 75.86 and 16.91 respectively. The average and standard deviation for her Class B of 37 students were 75.41 and 19.73 respectively. Using these results, test the claims at 5% level of significance.

(8 marks)

(d) A study was conducted to investigate some effects of physical training. A random sample of 10 trainer's weights before the training is shown in **Table Q3.1**, with all weights given in kilograms.

Table Q3.1 Weights of Trainers

99	57	62	69	74
77	59	92	70	85

Compute the 98% confidence interval of the mean weights of training.

(9 marks)

Q4 An experiment was held to investigate the relationship between the diameter of a nail and its maximum withdrawal strength, which measured in N/mm. **Table Q4.1** below shows the following results for 10 different diameters (in mm).

Table Q4.1 Diameter of a Nail and its Maximum Withdrawal Strength

Diameter	Strength
3.1	55
3.3	51
3.5	55
3.7	61
3.9	59
4.1	69
4.3	73
4.5	70
4.7	80
4.9	77

(a) Sketch a scatter plot of the data. Then, interpret the relationship between the variables.

(4 marks)

(b) Construct a linear regression model and interpret your results.

(11 marks)

(c) Based on appropriate coefficient, explain the relationship between the variables.

(6 marks)

(d) Define how good the model can explain the data, by using the appropriate coefficient.

(4 marks)

- END OF QUESTIONS -

APPENDIX A

TABLE OF FORMULA

Sampling Distributions:

$$\bar{X} \sim N(\mu, \sigma^2/n),\ Z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \sim N(0,1),\ T = \frac{\bar{x}-\mu}{s/\sqrt{n}},\ \bar{X}_1 - \bar{X}_2 \sim N\left(\mu_1-\mu_2, \frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}\right).$$

Estimations:

$$n = \left(\frac{Z_{\alpha/2}\cdot\sigma}{E}\right)^2, (\overline{x_1}-\overline{x_2}) - Z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}} < \mu_1-\mu_2 < (\overline{x_1}-\overline{x_2}) + Z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}},$$

$$(\overline{x_1}-\overline{x_2}) - Z_{\frac{\alpha}{2}}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}} < \mu_1-\mu_2 < (\overline{x_1}-\overline{x_2}) + Z_{\frac{\alpha}{2}}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}},$$

$$(\overline{x_1}-\overline{x_2}) - t_{\frac{\alpha}{2},v}\cdot S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} < \mu_1-\mu_2 < (\overline{x_1}-\overline{x_2}) + t_{\frac{\alpha}{2},v}\cdot S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

where Pooled estimate of variance, $S_p^2 = \frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}$ with $v = n_1+n_2-2$,

$$(\overline{x_1}-\overline{x_2}) - t_{\frac{\alpha}{2},v}\sqrt{\frac{1}{n}(s_1^2+s_2^2)} < \mu_1-\mu_2 < (\overline{x_1}-\overline{x_2}) + t_{\frac{\alpha}{2},v}\sqrt{\frac{1}{n}(s_1^2+s_2^2)}$$

with $v = 2(n-1)$

$$(\overline{x_1}-\overline{x_2}) - t_{\frac{\alpha}{2},v}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}} < \mu_1-\mu_2 < (\overline{x_1}-\overline{x_2}) + t_{\frac{\alpha}{2},v}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$$

with $v = \frac{\left(\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1-1}+\frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2-1}}$

$$\frac{(n-1)\cdot s^2}{\chi^2_{\alpha/2,v}} < \sigma^2 < \frac{(n-1)\cdot s^2}{\chi^2_{1-\alpha/2,v}},\ \text{with } v = n-1,$$

$$\frac{s_1^2}{s_2^2}\cdot\frac{1}{f_{\frac{\alpha}{2}}(v_1,v_2)} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{s_1^2}{s_2^2}\cdot f_{\frac{\alpha}{2}}(v_2,v_1)\ \text{with } v_1 = n_1-1 \text{ and } v_2 = n_2-1.$$

Hypothesis Testing:

$$Z = \frac{(\overline{X_1}-\overline{X_2})-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}, T = \frac{(\overline{X_1}-\overline{X_2})-(\mu_1-\mu_2)}{S_p\cdot\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}\ \text{with } v = n_1+n_2-2,$$

$$Z = \frac{(\overline{X_1}-\overline{X_2})-(\mu_1-\mu_2)}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}}, T = \frac{(\overline{X_1}-\overline{X_2})-(\mu_1-\mu_2)}{\sqrt{\frac{1}{n}(s_1^2+s_2^2)}}, T = \frac{(\overline{X_1}-\overline{X_2})-(\mu_1-\mu_2)}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}},$$

with $v = \frac{\left(\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1-1}+\frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2-1}}$; $S_p^2 = \frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}$.

Simple Linear Regressions:

$$S_{xy} = \sum x_i\, y_i - \frac{\sum x_i \cdot \sum y_i}{n}, S_{xx} = \sum {x_i}^2 - \frac{(\sum x_i)^2}{n}, S_{yy} = \sum {y_i}^2 - \frac{(\sum y_i)^2}{n}, x = \frac{\sum x}{n},$$

$$y = \frac{\sum y}{n}, \quad \hat{\beta}_1 = \frac{S_{xy}}{S_{xx}}, \quad \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}, \quad \hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x, \quad r = \frac{S_{xy}}{\sqrt{S_{xx} \cdot S_{yy}}},$$

$$SSE = S_{yy} - \hat{\beta}_1 S_{xy}, \quad MSE = \frac{SSE}{n-2}, \quad T = \frac{\hat{\beta}_1 - \beta_1^*}{\sqrt{\frac{MSE}{S_{xx}}}} \sim t_{n-2},$$

$$T = \frac{\hat{\beta}_0 - \beta_0^*}{\sqrt{MSE\left(\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}}\right)}} \sim t_{n-2}.$$