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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2023/2024

COURSE NAME	:	MATHEMATICS FOR ENGINEERING TECHNOLOGY II
COURSE CODE	:	BDJ 12303
PROGRAMME CODE	:	BDJ
EXAMINATION DATE	:	JULY 2024
DURATION	:	3 HOURS
INSTRUCTIONS	:	1. ANSWER ALL QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA ☐ Open book ☒ Closed book 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **FIVE (5)** PAGES

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Q1 Answer the following questions.

(a) Solve the following first-order homogeneous differential equations:

(i) $\frac{dy}{dx} = \frac{xy+y^2}{xy-x^2}$.

(5 marks)

(ii) $(2y + x)dy = (4y - x)dx.$

(5 marks)

(b) Solve the following first-order differential equations:

$$x\frac{dy}{dx} + 2y = \sin x.$$

(5 marks)

(c) Given an ordinary differential equation:

$$(2x + ye^{xy})dx + (\cos y + xe^{xy})dy = 0.$$

(i) Show that the given equation is exact.

(3 marks)

(ii) Hence, solve the exact equation.

(7 marks)

Q2 Answer the following questions.

(a) Determine the solution of the nonhomogeneous second-order differential equations by using the undetermined coefficients method.

$$y'' - 7y' + 6y = 36x, \qquad y(0) = 0,\ y'(0) = 4.$$

(10 marks)

(b) Determine the solution for the given differential equation by using the variation of parameters method.

$$y'' - 2y' + 2y = e^x(1 + \sin x).$$

(15 marks)

Q3 Answer the following questions.

(a) Determine the Laplace transform for each of the following functions:

(i) $f(t) = t^2e^{3t}$.

(5 marks)

(ii) $f(t) = \sin 3t \,.\cos 5t$.

Hint: $\{2\sin x \,.\cos y = \sin(x+y) + \sin(x-y)\}$

(5 marks)

(iii) $f(t) = (t+1)^3$.

(5 marks)

(b) Determine the inverse Laplace transform for each of the following functions:

(i) $f(s) = \frac{3s+8}{s^2+4}$.

(5 marks)

(ii) $f(s) = \frac{1}{s^2(s^2+4)}$.

(5 marks)

Q4 Answer the following questions.

(a) Given the initial-value problem (IVP), $y' = \frac{y+x}{xy}$, $y(0.4) = 1$.
Use Euler's method to obtain an approximate for y when $x = 1.4$, using a step size of $h = 0.2$.

(10 marks)

(b) Consider the following initial-value problem (IVP),

$$y' = \sqrt{x^2 + y}, \qquad y(0) = 0.8.$$

Solve for $0 \le x \le 0.8$, and $h = 0.4$ by using the fourth-order Runge-Kutta method.

(15 marks)

- END OF QUESTIONS -

APPENDIX A

FORMULA

Second-order Differential Equation

The roots of the characteristic equation and the general solution for the differential equation $ay'' + by' + cy = 0$

Characteristic equation: $am^2 + bm + c = 0.$		
Case	The roots of characteristic equation	General solution
1.	Real and different roots: m_1 and m_2	$y = Ae^{m_1 x} + Be^{m_2 x}$
2.	Real and equal roots: $m = m_1 = m_2$	$y = (A + Bx)e^{mx}$
3.	Complex roots: $m_1 = \alpha + \beta i$, $m_2 = \alpha - \beta i$	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

The method of undetermined coefficients

For non-homogeneous second-order differential equation $ay'' + by' + cy = \mathrm{f}(x)$ the particular solution is given by $y_p(x)$:

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + A_{n-1}x^{n-1} + \cdots + A_1 x + A_0$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(P\cos\beta x + Q\sin\beta x)$
$P_n(x)e^{\alpha x}$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)e^{\alpha x}$
$P_n(x)\begin{cases}\cos\beta x \\ \sin\beta x\end{cases}$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)\cos\beta x +$ $x^r(C_n x^n + C_{n-1}x^{n-1} + \cdots + C_1 x + C_0)\sin\beta x$
$Ce^{\alpha x}\begin{cases}\cos\beta x \\ \sin\beta x\end{cases}$	$x^r e^{\alpha x}(P\cos\beta x + Q\sin\beta x)$
$P_n(x)e^{\alpha x}\begin{cases}\cos\beta x \\ \sin\beta x\end{cases}$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)e^{\alpha x}\cos\beta x +$ $x^r(C_n x^n + C_{n-1}x^{n-1} + \cdots + C_1 x + C_0)e^{\alpha x}\sin\beta x$

Note : r is the least non-negative integer ($r = 0, 1, or\ 2$) which determines such that there are no terms in particular integral $y_p(x)$ corresponds to the complementary function $y_c(x)$.

The method of variation of parameters

If the solution of the homogeneous equation $ay'' + by' + cy = 0$ is $y_c = Ay_1 + By_2$, then the particular solution for $ay'' + by' + cy = \mathrm{f}(x)$ is $y = uy_1 + vy_2$

Where $u = -\int \frac{y_2 f(x)}{aW}dx + A$, $v = -\int \frac{y_1 f(x)}{aW}dx + B$, and $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_{21} y_1'$

Laplace Transform

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
e^{at}	$\frac{1}{s-a}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
$\sin at$	$\frac{a}{s^2+a^2}$	$\delta(t-a)$	e^{-as}
$\cos at$	$\frac{s}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$\int_0^t f(u)g(t-u)\,du$	$F(s)\cdot G(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$y(t)$	$Y(s)$
t^n, $n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	$y'(t)$	$sY(s)-y(0)$
$e^{at}f(t)$	$F(s-a)$	$y''(t)$	$s^2Y(s)-sy(0)-y'(0)$
$t^n f(t)$, $n=1,2,3,\ldots$	$(-1)^n\frac{d^n}{ds^n}F(s)$		

Euler's method

$$y_{i+1} = y_i + hy_i' = y_i + hf(x_i, y_i)$$

Fourth-order Runge-Kutta method

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

Which $k_1 = hf(x_i, y_i)$, $k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2})$,

$k_3 = hf(x_i + \frac{h}{2}, y_i + \frac{k_2}{2})$, $k_4 = hf(x_i + h, y_i + k_3)$.