Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2023/2024

COURSE NAME	:	MATHEMATICS I
COURSE CODE	:	BBP 10603
PROGRAMME CODE	:	BBA/BBE/BBF
EXAMINATION DATE	:	JULY 2024
DURATION	:	3 HOURS
INSTRUCTIONS	:	1. ANSWER ALL QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA ☐ Open book ☒ Closed book 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

Q1 (a) Calculate the value of x that satisfies the equation:

$$11^{3x} = \frac{1}{1331^{2(2x+1)}}$$

(5 marks)

(b) Solve $2\log_2 x - 6\log_x 8 = -9$.

(8 marks)

(c) Using de Moivre's theorem, compute $(1-i)^8$ in the form of $a+bi$.

(7 marks)

Q2 (a) Solve the quadratic equation:

(i) $5x^2 - 8x = 6$ by using quadratic formula.

(4 marks)

(ii) $3x^2 - 6x = 5$ by completing the square.

(5 marks)

(b) Express $\dfrac{4x-1}{(x+2)(x-2)}$ in the form of partial fraction.

(5 marks)

(c) Solve the inequality $\dfrac{3x+2}{(x+1)(x-1)} \geq 0$.

(6 marks)

Q3 (a) In the **Figure Q3(a)**, PQR is a straight line and PSQ forms a right-angled triangle. Given RQ = 10cm, PS = 24cm, and 5RQ = 2PQ.

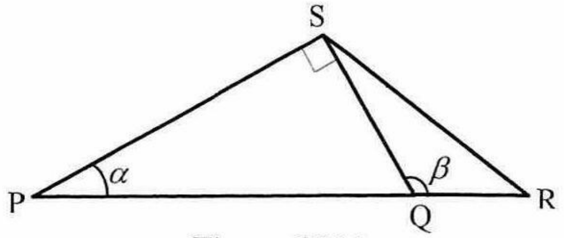

Figure Q3(a)

(i) Find the length of SQ.

(3 marks)

(ii) Determine the angle values of $\sin\alpha$.

(2 marks)

(iii) Calculate the angle of β.

(2 marks)

(b) Verify the identity:

$$\sec\theta - \tan\theta = \frac{\cos\theta}{1+\sin\theta}$$

(5 marks)

(c) Given $5\sin x + 12\cos x = r\cos(x+\alpha)$ for $0° \le x \le 360°$.

(i) Calculate the value of r and α.

(3 marks)

(ii) Hence, from your answer in **Q3(c)(i)**, solve $5\sin x + 12\cos x = 9$.

(5 marks)

Q4 (a) Given three (3) matrices; $J = \begin{bmatrix} 2 & 2 \\ 2x & 0 \\ -3 & x \end{bmatrix}$, $K = \begin{bmatrix} 0 & y \\ -1 & 7 \\ 2 & 11y \end{bmatrix}$, and $L = \begin{bmatrix} 2 & 0 \\ 9 & -7 \\ -5 & -7 \end{bmatrix}$.

(i) If $J - K = L$, determine the value of x and y.

(3 marks)

(ii) Then, compute the product of J^T and K.

(4 marks)

(b) Given that,

$$\begin{aligned} x + y + z &= 3 \\ 4x + 5y + 6z &= 24. \\ 3x + y - 2z &= 4 \end{aligned}$$

(i) Express the system of linear equations into the matrix equation $AX = B$.

(2 marks)

(ii) Find the determinant of matrix A.

(3 marks)

(iii) By using Gauss Jordan Elimination Method, determine the value of x, y and z. Do this following operation in order:
R_2-4R_1, R_3-3R_1, R_1-R_2, R_3+2R_2, R_1-R_3, -R_3, R_2-2R_3.

(8 marks)

Q5 (a) If $\mathbf{s} = 3i + 2j + 2k$ and $\mathbf{t} = 2i - 3j + 2k$, find $|2\mathbf{s} - 3\mathbf{t}|$.

(3 marks)

(b) Given $\mathbf{u} = 2i - j + 4k$, $\mathbf{v} = -6i + 2j - 12k$ and $\mathbf{w} = i - 2j + 5k$. Solve:

(i) $\mathbf{u} \bullet \mathbf{v}$.

(2 marks)

(ii) $\mathbf{v} \times \mathbf{w}$.

(3 marks)

(iii) $2\mathbf{v} + 4\mathbf{w} - \mathbf{u}$.

(3 marks)

(c) Find an equation of a line that passes through $Q(3,\ 1,\ 2)$ and $R(1,\ -2,\ -2)$.

(3 marks)

(d) Find the vector equation of the plane that containing points $A(1,\ 0,\ 2)$, $B(-2,\ 1,\ 0)$, and $C(3,\ 1,\ -1)$.

(6 marks)

– END OF QUESTIONS –

APPENDIX A

Formula

APPENDIX 1 Real number

$$z = a + bi$$
$$\bar{z} = a - bi$$
$$z = r(\cos\theta + i\sin\theta)$$

$$|z| = r = \sqrt{a^2 + b^2}$$
$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

$$z_1 z_2 = r_1 r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$$

$$z^{\frac{1}{n}} = r^{\frac{1}{n}} e^{\left(\frac{\theta + 2k\pi}{n}\right) i}$$

$$z^n = r^n\left[\cos n\theta + i\sin n\theta\right]$$

$$z^{\frac{1}{n}} = r^{\frac{1}{n}}\left(\cos\frac{\theta + 2k\pi}{n} + i\sin\frac{\theta + 2k\pi}{n}\right)$$

$$\log_a x = \frac{\log_a x}{\log_a b}$$

APPENDIX 2 Polynomials

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \quad x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c, \quad x_{i+2} = \frac{x_i f(x_{i+1}) - x_{i+1} f(x_i)}{f(x_{i+1}) - f(x_i)}$$

$$\frac{P(x)}{(a_1x + b_1)(a_2x + b_2)\ldots(a_kx + b_k)} = \frac{A_1}{a_1x + b_1} + \frac{A_2}{a_2x + b_2} + \ldots + \frac{A_k}{a_kx + b_k}$$

$$\frac{P(x)}{(ax + b)^k} = \frac{A_1}{ax + b} + \frac{A_2}{(ax + b)^2} + \ldots + \frac{A_k}{(ax + b)^k}$$

$$\frac{P(x)}{ax^2 + bx + c} = \frac{Ax + B}{ax^2 + bx + c}$$

$$\frac{P(x)}{(ax^2 + bx + c)^k} = \frac{Ax + B}{ax^2 + bx + c} + \frac{Ax + B}{(ax^2 + bx + c)^2} + \ldots + \frac{Ax + B}{(ax^2 + bx + c)^k}$$

APPENDIX 3 Trigonometry

$$\sin(-\theta) = -\sin\theta$$
$$\cos(-\theta) = \cos\theta$$
$$\tan(-\theta) = -\tan\theta$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$\csc\theta = \frac{1}{\sin\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$
$$\sec\theta = \frac{1}{\cos\theta}$$

$\sin^2 x + \cos^2 x = 1$, $\tan^2 x + 1 = \sec^2 x$, $1 + \cot^2 x = \csc^2 x$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta} \qquad \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\sin 2\theta = 2\sin\theta\cos\theta, \quad \cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}, \quad \cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}, \quad \tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

$a\sin\theta + b\cos\theta = r\sin(\theta+\alpha) = r(\sin\theta\cos\alpha + \cos\theta\sin\alpha) = (r\cos\alpha)\sin\theta + (r\sin\alpha)\cos\theta$ and

$a = r\cos\alpha$ and $b = r\sin\alpha$

$$x_1^{(k+1)} = \frac{b_1 - a_{12}x_2^{(k)} - a_{13}x_3^{(k)}}{a_{11}}, \quad x_2^{(k+1)} = \frac{b_2 - a_{21}x_1^{(k+1)} - a_{23}x_3^{(k)}}{a_{22}}, \quad x_3^{(k+1)} = \frac{b_3 - a_{31}x_1^{(k+1)} - a_{32}x_2^{(k+1)}}{a_{33}}$$

APPENDIX 4 Matrices

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \quad |A| = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}, \quad A^{-1} = \frac{1}{|A|}Adj\ A$$

$$X = A^{-1}B, \qquad Adj\ A = \begin{pmatrix} +M_{11} & -M_{12} & +M_{13} \\ -M_{21} & +M_{22} & -M_{23} \\ +M_{31} & -M_{32} & +M_{33} \end{pmatrix}^T$$

APPENDIX 5 Vector

$$|\mathbf{u}| = \sqrt{a^2 + b^2 + c^2} \qquad \hat{\mathbf{u}} = \frac{\mathbf{u}}{|\mathbf{u}|}$$

$$\mathbf{u} \bullet \mathbf{v} = u_1v_1 + u_2v_2 + u_3v_3 \qquad \mathbf{u} \bullet \mathbf{v} = |\mathbf{u}||\mathbf{v}|\cos\theta$$

$$\theta = \cos^{-1}\left(\frac{\mathbf{u} \bullet \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}\right) \qquad A = \frac{1}{2}|\mathbf{u} \times \mathbf{v}|$$

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} \qquad \frac{x - x_0}{a_1} = \frac{y - y_0}{a_2} = \frac{z - z_0}{a_3}$$

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$