UTHM
Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	:	TECHNICAL MATHEMATICS I
COURSE CODE	:	DAS 11003
PROGRAMME CODE	:	DAK
EXAMINATION DATE	:	DECEMBER 2019 / JANUARY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS IN **SECTION A** AND **THREE (3)** QUESTIONS IN **SECTION B**

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

SECTION A

Q1 (a) Given the matrices;

$$A=\begin{bmatrix}-4 & 3 & 4\\ 12 & -9 & -11\\ -1 & 1 & 2\end{bmatrix}, B=\begin{bmatrix}2 & -3 & 1\\ 0 & 2 & 4\\ 7 & -7 & 2\end{bmatrix} \text{ and } C=\begin{bmatrix}-11 & 9 & -13\\ 14 & 0 & 7\\ 6 & -3 & 9\end{bmatrix}.$$

Find the values of

(i) $A^T - 3B$.

(3 marks)

(ii) BC.

(4 marks)

(b) Given;

$$\begin{aligned} x+y+z&=50\\ 2x-z&=1\\ x-3y+z&=-10 \end{aligned}.$$

(i) Write the matrix equation $AX=B$ of the system equation.

(1 mark)

(ii) Write the augmented matrix, $[A|B]$.

(1 mark)

(iii) Find the determinant of matrix A.

(2 marks)

(iv) Solve the above system for x, y and z by using Gauss-Jordan elimination method. Do the following operations in order:

$-2R_1+R_2,\ -R_1+R_3,\ -\frac{1}{2}R_2,\ -R_2+R_1,\ 4R_2+R_3,\ \frac{1}{6}R_3,\ \frac{1}{2}R_3+R_1,$
$-\frac{3}{2}R_3+R_2.$

(9 marks)

Q2 (a) Express $\dfrac{x^4}{(x-1)(x+2)}$ in the form of partial fractions.

(10 marks)

(b) Find the root of the $f(x)=x^6-x-1$ in the interval $[1,2]$ using Bisection method. Iterate until $|f(x_i)|<\varepsilon=0.005$. Show your calculation in three decimal places.

(10 marks)

SECTION B

Q3 (a) Solve $\dfrac{1}{e^{-4x^2}}=\dfrac{e^{21x}}{e^{-18}}$.

(4 marks)

(b) Solve $\sqrt{x}+\sqrt{x+4}=2$.

(6 marks)

(c) Solve the following simultaneous equations;

$$\log_{10}(x-2)+\log 2=2\log_{10}y.$$
$$\log_{10}(x-3y+3)=0.$$

(10 marks)

Q4 (a) Find the sum of $\sum_{r=1}^{20}(2r-1)^2$.

(5 marks)

(b) Find the common difference of an arithmetic sequence if the sum of the first n^{th} term, $S_n=\dfrac{n}{4}(3n+13)$.

(7 marks)

(c) Given the geometric series $2+6+18+\ldots.$

(i) Calculate the sum of the first 9 terms of the geometric series.

(3 marks)

(ii) Find the smallest number of terms that should be taken so that the sum exceeds 100 000.

(5 marks)

Q5 (a) (i) Prove the identity $\dfrac{2}{\operatorname{cosec}\theta-1}-\dfrac{2}{\operatorname{cosec}\theta+1}=4\tan^2\theta.$

(5 marks)

(ii) Solve the trigonometric equation;

$$4\cos\left(\theta-30^\circ\right)=3\sin\left(\theta+60^\circ\right);\quad 0^\circ\leq\theta\leq360^\circ.$$

(6 marks)

(b) If $\sin\theta=\dfrac{3}{5}$ and $0^\circ<\theta<90^\circ$, evaluate $\tan\dfrac{\theta}{2}$.

(3 marks)

(c) Solve $\cos 2\theta=0;\quad 0^\circ<\theta<360^\circ.$

(6 marks)

Q6 (a) Solve the inequality $\dfrac{(x+1)(x-2)}{(1-2x)}\geq 0$.

(7 marks)

(b) Show that $\left(p-p^{-1}\right)\left(p^{\frac{4}{3}}+p^{-\frac{2}{3}}\right)=\dfrac{p^2-p^{-2}}{p^{-\frac{1}{3}}}.$

(6 marks)

(c) Show that $\dfrac{\sec\theta}{\sec\theta+\operatorname{cosec}\theta}=\dfrac{\tan\theta}{\tan\theta+1}.$

(7 marks)

– END OF QUESTIONS –

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FORMULA

Polynomials

$\log_a x = \dfrac{\log_a x}{\log_a b}$ $\quad a^3 - b^3 = (a-b)(a^2+ab+b^2)$ $\quad a^3 + b^3 = (a+b)(a^2-ab+b^2)$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$, $\quad x^2+bx+c = \left(x+\dfrac{b}{2}\right)^2 - \left(\dfrac{b}{2}\right)^2 + c$, $\quad x_{i+2} = \dfrac{x_i f(x_{i+1}) - x_{i+1} f(x_i)}{f(x_{i+1}) - f(x_i)}$

Sequence and Series

$\sum_{k=1}^{n} c = cn$, $\quad \sum_{k=1}^{n} k = \dfrac{n(n+1)}{2}$, $\quad \sum_{k=1}^{n} k^2 = \dfrac{n(n+1)(2n+1)}{6}$, $\quad \sum_{k=1}^{n} k^3 = \left(\dfrac{n(n+1)}{2}\right)^2$

$x_n = a + (n-1)d \quad S_n = \dfrac{n}{2}[2a+(n-1)d]$, $\quad S_n = \dfrac{n}{2}(a+u_n)$

$x_n = ar^{n-1}$, $\quad S_n = \dfrac{a(r^n-1)}{r-1}, r>1$ **OR** $S_n = \dfrac{a(1-r^n)}{1-r}, r<1$, $\quad S_\infty = \dfrac{a}{1-r}$.

$x_n = S_n - S_{n-1}$

$(1+b)^n = 1 + nb + \dfrac{n(n-1)}{2!}b^2 + \dfrac{n(n-1)(n-2)}{3!}b^3 + \ldots\ldots$

Trigonometry

$\sin^2 x + \cos^2 x = 1$, $\quad \tan^2 x + 1 = \sec^2 x$, $\quad 1 + \cot^2 x = \csc^2 x$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ $\quad \cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$ $\quad \tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

$\sin 2\theta = 2\sin\theta\cos\theta$, $\quad \cos 2\theta = \cos^2\theta - \sin^2\theta$
$= 2\cos^2\theta - 1$
$= 1 - 2\sin^2\theta$

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FORMULA

$$\sin\frac{\theta}{2}=\pm\sqrt{\frac{1-\cos\theta}{2}},\quad \cos\frac{\theta}{2}=\pm\sqrt{\frac{1+\cos\theta}{2}},\quad \tan\frac{\theta}{2}=\pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

$a\sin\theta+b\cos\theta=r\sin(\theta+\alpha)=r(\sin\theta\cos\alpha+\cos\theta\sin\alpha)=(r\cos\alpha)\sin\theta+(r\sin\alpha)\cos\theta$ and

$a=r\cos\alpha$ and $b=r\sin\alpha$

$$x_1^{(k+1)}=\frac{b_1-a_{12}x_2^{(k)}-a_{13}x_3^{(k)}}{a_{11}},\quad x_2^{(k+1)}=\frac{b_2-a_{21}x_1^{(k+1)}-a_{23}x_3^{(k)}}{a_{22}},\quad x_3^{(k+1)}=\frac{b_3-a_{31}x_1^{(k+1)}-a_{32}x_2^{(k+1)}}{a_{33}}$$

Matrices

$$A=\begin{pmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{pmatrix},\ |A|=a_{11}\begin{vmatrix}a_{22}&a_{23}\\a_{32}&a_{33}\end{vmatrix}-a_{12}\begin{vmatrix}a_{21}&a_{23}\\a_{31}&a_{33}\end{vmatrix}+a_{13}\begin{vmatrix}a_{21}&a_{22}\\a_{31}&a_{32}\end{vmatrix}$$