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FINAL EXAMINATION SEMESTER I SESSION 2009/2010

THIS EXAMINATION PAPER CONSISTS OF **F PAGES**

PARTA

 $Q1$ It is believed that the amount of nitrogen fertilizer used per acre has a direct effect on the amount of wheat produced. The SPSS output for 12 sample of fertilizer used per test plot and the amount of wheat harvested is shown in Figure Q1.

Figure Q1: The Wheat Harvested vs Nitrogen Fertilizer

Refer to the given SPSS output and answer the following questions.

 (a) From the graph in Figure $Q1$, describe the relationship between harvested and nitrogen fertilizer.

 (2 marks)

- (b) Identify the coefficient of determination, R^2 and the correlation coefficient, *r*. Describe your answers. (4 marks)
- (c) State the regression model. Then interpret the result.

(2 marks)

(2 marks)

- (d) Estimate the amount of wheat harvested per test plot when the amount of nitrogen fertilizer used per test plot is 100 kg.
- (e) Test the hypothesis that the intercept is less than 0 at significance level 0.01. Describe your conclusion.

(5 marks)

(f) Test the hypothesis that the slope is not equal to 0 at significance level 0.05. Describe your conclusion.

(5 marks)

Q2 (a) A process is supposed to produce washer with mean inner diameter of 25 mm. A random sample of **11** washers produced by this process shows in the **Table Q2 (a)** below:

Assuming the inner diameter of all washers produced by this process follow a normal distribution. Can we conclude that the variance of the inner diameter for all washers produced by this process is not less than 0.04 at significance level 0.01?

(9 marks)

(b) Before the development of UTHM at Parit Raja, a random sample of 11 single storey houses in the vicinity of UTHM showed a mean value of RM 90,000 with standard deviation of RM 5,000. After the development of UTHM, a random sample of 17 single storey houses in the vicinity of UTHM showed a mean value of RM 110,000 with standard deviation of RM 8,000. Test the hypotheses that the development of UTHM increased the mean value of single storey houses in the vicinity of UTHM by more than RM 25,000 at significance level 0.05. Assume the population variances are equal.

(11 marks)

PART B

Q3 Let *X* is a random variable with probability density function (pdf) given as:

$$
f(x) = \begin{cases} \frac{1}{6}(x+\frac{1}{2}), & 0 \le x \le 3, \\ 0, & \text{others.} \end{cases}
$$

 \mathbf{c} (a) Show that $\int f(x)dx$

(2 marks)

- (b) Find *P(X> 1).* (3 marks)
- (c) Find *F(X).* (5 marks)
- (d) Calculate the expected value of *X* and variance of *X.*

(e) Calculate
$$
Var(\frac{1}{2}X+1)
$$
.

(2 marks)

(8 marks)

- **Q4** (a) Scores on examination are assumed to be normally distributed with a mean of 72 and a variance of 50.
	- (i) What is the probability of a person taking the exam and get score between 70 and 76?
	- (ii) If known that 15% of the students get grade A, find the minimum score a student must achieve to earn grade A?

(7 marks)

- (b) The probability that a single radar set will detect an enemy plane is 0.85. If we have five radar sets, which independently of each other, find the probability that
	- (i) exactly four sets will detect the plane.
	- (ii) at least one set will detect the plane.

(6 marks)

- (c) In a very large industrial company, the average number of fatal accidents per month is 0.3. Find the probability of each of the following event.
	- (i) No fatal accident in five months.
	- (ii) At most 7 fatal accidents in a year.

(7 marks)

Q5 (a) Children like to eat chocolate cookies with chocolate chips. One day mother makes the cookies and the chocolate chips were counted. Let *X* is a random number of chocolate chips in the cookies. **Table Q5 (a)** below shows the probability distributions function *ofX.*

Table Q5 (a) : Probability Distributions Function of *X*

Find the

- (i) population mean and variance.
- (ii) mean and variance for 40 samples of cookies.
- (iii) probability of the average number of chocolate chips is less than 4.5 in 40 samples of cookies.

(9 marks)

- (b) Two type of rod made from alloy, Rod *A* and Rod *B* are being investigated to know their breaking strength. It was found that the mean and variance of breaking strength (in kilograms) are normally distributed $X_A \sim N(202, 4)$ and $X_B \sim N(200, 3)$. A sample of 13 Rod *A* and 15 Rod *B* are selected at random. Find the probability that
	- (i) the mean breaking strength for Rod *A* is greater than 201.3 kg.
	- (ii) the mean breaking strength of Rod *B* is lower than Rod *A.*
	- (iii) the difference between mean breaking strength Rod *A* and Rod *B* are at least 0.2 kg-

(11 marks)

Q6 (a) A postman wants to estimate the population standard deviation for waiting times at certain counters. A random sample of $n = 10$ times (the number of minutes people have to wait) are as shown in **Table Q6 (a):**

Construct a 98% confidence interval for the population standard deviation.

(7 marks)

(b) A researcher is concern that the variability of responses using two different experimental procedures may not be the same. Before conducting his research, he conducts a prestudy with random samples of 11 and 9 from the two experimental procedures and gets $s_1^2 = 7.14$ and $s_2^2 = 3.21$ respectively. Construct a 95% confidence interval for the ratio of the two variances.

(6 marks)

(c) The qualities of two types of automobile tires were compared by road-testing samples of $n_1 = 15$ and $n_2 = 17$ tires for each type. The number of miles until wearout was defined as a specific amount of tire wear. The test results are given in the **Table Q6 (c)** below:

Table Q6 (c): Test Result for Automobile Tyres

Tire 1	Tire 2	
Mean: $26,400$ miles	Mean: $25,100$ miles	
Sample variance: 1200	Sample variance: 1400	

Construct a 90% confidence intervals for the difference mean between Tire 1 and Tire 2. Assume that the distribution is normal with unequal variances.

(7 marks)

FINAL EXAMINATION

SEMESTER / SESSION: SEM I / 2009/2010 COURSE: 3 BEP / 3 BER / 3 BET / 3 BEM / 3 BEE / 3 BFF / 3 BFA 3 / BFB / 3 BFP
BSM 2922 SUBJECT : ENGINEERING STATISTICS CODE: FOR

Formulae Random Variable : 00 $\sum P(x_i) = 1$, $E(X) = \sum x \cdot P(x)$, $E(X^2) = \sum x^2 \cdot P(x)$, $f(x) dx = 1$, $E(X) = \int_0^x x \cdot P(x) dx$, $E(X^2) = \int_0^x x^2 \cdot P(x) dx$, $Var(X) = E(X^2) - [E(X)]^2$. Special Probability Distributions : $P(x = r) = {}^{n}C_{r} \cdot p^{r} \cdot q^{n-r}, r = 0, 1, ..., n, X \sim B(n, p), P(X = r) = \frac{e^{-\mu} \cdot \mu^{r}}{r!}, r = 0, 1, ..., \infty,$ $X \sim P_0(\mu)$, $Z = \frac{T}{\mu}$, $Z \sim N(0, 1)$, $X \sim N(\mu, \sigma^2)$. Sampling Distributions : $\overline{X} \sim N(\mu, \sigma^2/n), Z = \frac{A-\mu}{I} \sim N(0, 1), T = \overline{Y}$ σ/\sqrt{n} *s/* \sqrt{n} $-\overline{X_2} \sim N \left[\mu_1 - \mu_2, \frac{\sigma_1}{\sigma_2} + \frac{\sigma_2}{\sigma_2} \right]$ Estimations : (7) $n = \frac{Z_{\alpha/2}}{Z}$ $\left(\begin{array}{cc} E \end{array} \right)$ \prime \rightarrow $x_1 - x_2 +$ $\left(\bar{x}_1 - \bar{x}_2\right) - Z_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < \left(\bar{x}_1 - \bar{x}_2\right)$ $($ $x_1 - x_2 + \left[x_1 - x_2 \right] - t_{\alpha/2,\nu} \cdot S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} < \mu_1 - \mu_2 < \left[1 \right]$ $\sqrt{2}$ $x_1 - x_2$ \vee / $- Z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \cdot \mu_1 - \mu_2 <$ $x_1 - x_2$ \vee \vee $x_1 - x_2$ \vee **" 2 2** \prime \rightarrow \prime x_1-x_2 \vee y where Pooled estimate of variance, $S_n^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1)s_2^2}$ with $v = n_1 + n_2 - 2$, $1 \quad 1$ $+n_{2} - 2$ $x_1 - x_2$ $\sqrt{2}$ x_1-x_2 \vee y $\left(\frac{1}{n}\left(s_1^2 + s_2^2\right) < \mu_1 - \mu_2 < \left(\bar{x}_1 - \bar{x}_2\right)\right)$ $+ i_{\alpha/2,\nu} \sqrt{-\Psi_1 + \Psi_2}$ with $\nu = 2(n-1)$, $\int_{a^2} 2 \cdot \cdot \cdot \cdot$ ² $- t_{a/2, y} \left| \frac{s_1^2}{s_1^2} + \frac{s_2^2}{s_2^2} \right|$ $a/2, v_1$ $\frac{1}{2} + \frac{1}{2} < \mu_1 - \mu_2 <$ $\sqrt{n_1}$ n_2 *(- * x_1 – \vee) $\frac{3}{2}$ $\frac{3}{2}$ $\frac{1}{2}$ + $\iota_{\alpha/2,\nu} \sqrt{n_1 + n_2}$ with $\nu = \frac{1}{\left(\frac{s_1^2}{n_1}\right)^2}$ $\langle n_1 \rangle$ $n_{\rm s} - 1$ $v < \sigma^2 < \frac{(n-1)^{3}}{2}$ with $v = n-1$ $\lambda a/2, \nu$ λ $\lambda - a$

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$$
\frac{s_1^2}{s_2^2} \cdot \frac{1}{f_{\alpha/2}(v_1, v_2)} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{s_1^2}{s_2^2} \cdot f_{\alpha/2}(v_2, v_1) \text{ with } v_1 = n_1 - 1 \text{ and } v_2 = n_2 - 1.
$$
\nHypothesis Testings :
\n
$$
Z = \frac{(\overline{X_1} - \overline{X_2}) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \quad S_p \cdot \frac{1}{\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}
$$
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$$
Z = \frac{(\overline{X_1} - \overline{X_2}) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \quad S_p \cdot \frac{1}{\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}
$$
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$$
Z = \frac{(\overline{X_1} - \overline{X_2}) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \quad \sqrt{\frac{1}{n_1}(\varsigma_1^2 + \varsigma_2^2)} \quad T = \frac{(\overline{X_1} - \overline{X_2}) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}
$$
\n
$$
v = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\left(\frac{s_1^2}{n_1}\right)^2 - \left(\frac{s_2^2}{n_2}\right)^2}; \quad S_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}; \quad \chi^2 = \frac{(n - 1)s^2}{\sigma^2}
$$
\n
$$
S_{xy} = \sum x_i y_i - \frac{\sum x_i \cdot \sum y_i}{n_1}, \quad S_{xy} = \sum y_i^2 - \frac{(\sum y_i)^2}{n_1}, \quad S_x = \sum x_i \cdot \frac{\sum y_i}{n_2}, \quad S_x = \sum y_i \cdot \frac
$$

$$
T = \frac{\hat{\beta}_1 - \beta_1^*}{\sqrt{\frac{MSE}{S_{xx}}}} \sim t_{n-2}, \ T = \frac{\hat{\beta}_0 - \beta_0^*}{\sqrt{\frac{MSE}{n} \left(\frac{1}{n} + \frac{\bar{x}^*}{S_{xx}}\right)}} \sim t_{n-2}.
$$

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