UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2009/2010

SUBJECT	:	MATHEMATICS II
CODE	:	BSM 1253
COURSE	:	1 BBV
DATE	:	APRIL 2010
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER ALL QUESTIONS

THIS EXAMINATION PAPER CONSISTS OF 5 PAGES

Q1 (a) Find the domain of the function $f(x) = \ln\sqrt{\dfrac{x^2-5x+4}{x-2}}$.

(4 marks)

(b) Given functions $f(x) = x^2$ and $g(x) = \sqrt{x-6}$.

(i) Find $(f \circ f \circ g)(x)$ and $(g \circ g \circ f)(x)$.

(ii) Is the composition of function $(f \circ g)(x) = (g \circ f)(x)$?

(6 marks)

(c) Let

$$f(x) = \begin{cases} kx^2, & x \le 2, \\ 2x+k, & x > 2. \end{cases}$$

Find the value of k that will make the function f is continuous everywhere.

(4 marks)

(d) Given that $f(x) = \dfrac{x^2-4}{x^2-x-2}$.

(i) Find the value of x at which f is discontinuous.

(ii) How to define the function of f (if possible) such that f is continuous everywhere?

(6 marks)

Q2 (a) Given that $f(x) = \sqrt{x}$. Find $\lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$.

(4 marks)

(b) Find $\lim\limits_{x\to\infty} x^2 \sin\dfrac{1}{x}\tan\dfrac{5}{x}$.

(4 marks)

(c) (i) Find $\dfrac{dy}{dx}$ for $y = \cos\left(\sqrt{1+\sin x}\right)$.

(ii) Find $\dfrac{d\omega}{d\lambda}$ for $a^2\omega^2 + b^2\lambda^2 = c$, where a, b and c are constant.

(iii) Find $\dfrac{dy}{dx}$ for parametric equations $x = r\cos\theta$ and $y = r\sin\theta$, where r is constant and θ is parameter.

(12 marks)

Q3 (a) Show that both of functions $f(x)=(x-1)^4$ and $g(x)=x^3-3x^2+3x-2$ have stationary point at $x=1$.

(4 marks)

(b) Use both the first and the second derivative test to show that $f(x)=x^3-3x+3$ has a relative minimum at $x=1$ and a relative maximum at $x=-1$.

(4 marks)

(c) Evaluate the following integrals by using suitable technique of integrations.

(i) $\int(1+\sin t)^9\cos t\,dt$.

(ii) $\int\dfrac{x^4+5x^2+2}{x^3+2x}dx$.

(iii) $\displaystyle\int_0^1 x^2e^{2x}dx$.

(12 marks)

Q4 (a) Find the area of the region enclosed by the curves $y=-x$, $y=\sqrt{2-x}$ and x-axis.

(7 marks)

(b) Find the volume of the solid that results when the region enclosed by the curves $y=x$ and $y=2-x^2$ is revolved about the x-axis.

(7 marks)

(c) Use the integral to show that the total arc length of the circle $x=3\cos\theta$, $y=3\sin\theta$ $(0\le\theta\le2\pi)$ is 6π.

(6 marks)

Q5 (a) Determine whether the series below converges or diverges using the suitable test.

(i) $\displaystyle\sum_{k=1}^{\infty}\frac{4k^2-2k+6}{8k^7+k-8}$.

(ii) $\displaystyle\sum_{k=1}^{\infty}\frac{k!}{k^3}$.

(12 marks)

(b) Given the sequence $\frac{1}{2}, \frac{3}{5}, \frac{5}{8}, \frac{7}{11}, \ldots$

(i) Find a formula for the general term of the sequence starting with $n = 1$.

(ii) Determine whether the sequence converges, and if so find its limit.

(8 marks)