UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2009/2010

SUBJECT	:	MATHEMATICS II
CODE	:	DSM 1923
COURSE	:	1 DDM / DDT / DFA / DFT 2 DDM / DDT
DATE	:	APRIL 2010
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER ALL QUESTIONS IN **PART A** AND **THREE (3)** QUESTIONS IN **PART B**

THIS EXAMINATION PAPER CONSISTS OF 8 PAGES

PART A

Q1 (a) Determine whether the following integrations are improper or proper integral. Give your reason.

(i) $\int_1^5 \frac{dx}{\sqrt{x-3}}$.

(ii) $\int_1^e \frac{\ln x}{\sqrt{x}}\, dx$.

(iii) $\int_1^\infty \frac{dx}{x^2}$.

(6 marks)

(b) Determine whether the following integrations converge or diverge. If converge, what is the value?

(i) $\int_{-1}^1 \frac{1}{x^{\frac{4}{3}}}\, dx$.

(ii) $\int_0^\infty \frac{x}{\left(x^2+1\right)^2}\, dx$.

(14 marks)

Q2 (a) Curve of $y=2x^2+10$ and line $y=4x+16$ intersects at point A and B. Determine

(i) points A and B.
(ii) the area between the curve and the line.

(8 marks)

(b) Determine the volume of revolution of the solid formed by rotating the line $y=3x-2$ about the y-axis between $y=0$ and $y=5$.

(5 marks)

(c) Approximate $\int_1^2 \frac{\ln x}{1+x}\, dx$ using Simpson's rule with 10 intervals. (Round your answer in 3 decimal places).

(7 marks)

PART B

Q3 (a) Determine the continuity of $f(x)=\begin{cases} 9x-2, & x\le 1 \\ 7x^2, & x>1 \end{cases}$ at $x=1$.

(5 marks)

(b) Compute

(i) $\lim_{x\to 2}\frac{\sqrt{4x^2+9}}{x^2-8}$.

(ii) $\lim_{x\to 1}\frac{\sqrt{2x+1}-\sqrt{3}}{x-1}$.

(iii) $\lim_{x\to 0}\frac{\sin^2 x}{3x^2}$.

(8 marks)

(c) Use the L'Hopital's rule to compute

(i) $\lim_{x\to 0}\frac{\sin 2x}{x}$.

(ii) $\lim_{x\to \infty} xe^{-2x}$.

(7 marks)

Q4 (a) Find $\frac{dy}{dx}$ of the given functions.

(i) $y=7x^2e^{-x^2}$.

(ii) $x^2\sin y+2x=y^2$.

(iii) $y=\frac{\sqrt[3]{x+1}}{(x+2)\sqrt{x+3}}$. (Hint: Use natural logarithm on both sides)

(12 marks)

(b) A rectangular storage area is to be constructed along the side of a tall building. A security fence is required along the remaining 3 sides of the area **(Figure Q4(b))**. What is the maximum area that can be enclosed with 800 m of fencing?

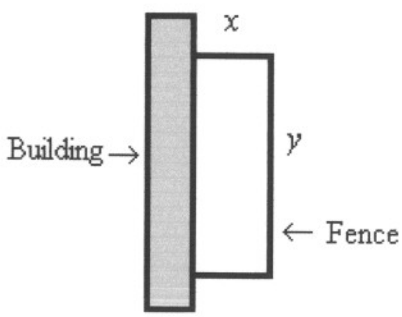

Figure Q4(b)

(8 marks)

Q5 (a) Evaluate the given integral.

(i) $\int e^{5x}\left(\frac{e^{2x}}{7}+\frac{3}{e^{3x}}\right)dx$.

(ii) $\int \frac{\ln x}{x^5}\,dx$. (Hint: Use by part integration)

(iii) $\int \frac{3x+2}{\sqrt{x-9}}\,dx$. (Hint: Use substitution method)

(12 marks)

(b) (i) Express $\frac{x^2+x-1}{x(x^2-1)}$ in the form of partial fraction.

(ii) Hence, evaluate $\int \frac{x^2+x-1}{x(x^2-1)}\,dx$.

(8 marks)

Q6 (a) Using tabular method to evaluate

(i) $\int x^3 e^{2x}\,dx$.

(ii) $\int_0^{\frac{\pi}{6}} \cos 5x \sin 4x\,dx$.

(13 marks)

(b) Calculate the arc length of $y = 2x^{\frac{3}{2}} + 3$ from $x = 1$ to $x = 4$. (Round your answer in 4 decimal places).

(7 marks)

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FORMULAE

DIFFERENTIATION

$\frac{d}{dx}[ax]=a$	$\frac{d}{dx}[\sin ax]=a\cos ax$
$\frac{d}{dx}[x^n]=nx^{n-1}$	$\frac{d}{dx}[\cos ax]=-a\sin ax$
$\frac{d}{dx}[c\ f(x)]=c\ \frac{d}{dx}[f(x)]$	$\frac{d}{dx}[\tan ax]=a\sec^2 ax$
$\frac{d}{dx}[f(x)\pm g(x)]=\frac{d}{dx}[f(x)]\pm\frac{d}{dx}[g(x)]$	$\frac{d}{dx}[\sec ax]=a\sec ax\tan ax$
$\frac{d}{dx}[f(x)g(x)]=f(x)\cdot\frac{d}{dx}[g(x)]+g(x)\cdot\frac{d}{dx}[f(x)]$	$\frac{d}{dx}[\cot ax]=-a\csc^2 ax$
$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)\cdot\frac{d}{dx}[f(x)]-f(x)\cdot\frac{d}{dx}[g(x)]}{[g(x)]^2}$	$\frac{d}{dx}[\csc ax]=-a\csc ax\cot ax$
$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$	$\frac{d}{dx}[\sin^{-1}ax]=\frac{1}{\sqrt{1-a^2x^2}}\frac{d}{dx}[ax]$
$\frac{d}{dx}[e^{ax}]=ae^{ax}$	$\frac{d}{dx}[\cos^{-1}ax]=\frac{-1}{\sqrt{1-a^2x^2}}\frac{d}{dx}[ax]$
$\frac{d}{dx}[a^x]=a^x\ln a$	$\frac{d}{dx}[\tan^{-1}ax]=\frac{1}{1+a^2x^2}\frac{d}{dx}[ax]$
$\frac{d}{dx}\ln\lvert ax+b\rvert=\frac{a}{ax+b}$	$\frac{d}{dx}[\cot^{-1}ax]=\frac{-1}{1+a^2x^2}\frac{d}{dx}[ax]$
$\frac{d}{dx}[\log_a u]=\frac{1}{u}\log_a e\frac{d}{dx}[u(x)]$	$\frac{d}{dx}[\sec^{-1}ax]=\frac{1}{\lvert ax\rvert\sqrt{a^2x^2-1}}\frac{d}{dx}[ax]$

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INTEGRATION

$$\int c\, f(x)dx = c\, F(x) + C$$

$$\int [f(x) \pm g(x)]\, dx = F(x) \pm G(x) + C$$

$$\int x^n\, dx = \frac{x^{n+1}}{n+1} + C\,, (n \neq -1)$$

$$\int u\, dv = uv - \int v\, du$$

$$\int \frac{1}{ax+b}\, dx = \frac{1}{a}\ln|ax+b| + C$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C$$

$$\int \sin(ax)\, dx = -\frac{\cos(ax)}{a} + C$$

$$\int \cos(ax)\, dx = \frac{\sin(ax)}{a} + C$$

$$\int \tan(ax)\, dx = \ln|\sec(ax)| + C$$

$$\int \sec^2(ax)\, dx = \frac{\tan(ax)}{a} + C$$

$$\int \csc^2(ax)\, dx = -\frac{\cot(ax)}{a} + C$$

$$\int \sec(ax)\tan(ax)\, dx = \frac{\sec(ax)}{a} + C$$

$$\int \csc(ax)\cot(ax)\, dx = -\frac{\csc(ax)}{a} + C$$

$$\int \csc(ax)\, dx = -\ln|\csc(ax) + \cot(ax)| + C$$

$$\int \sec(ax)\, dx = \ln|\sec(ax) + \tan(ax)| + C$$

$$\int \frac{1}{\sqrt{a^2 - u^2}}\, du = \sin^{-1}\left(\frac{u}{a}\right) + C\ ;\ u^2 < a^2$$

$$\int \frac{1}{\sqrt{a^2 + u^2}}\, du = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \frac{1}{u\sqrt{u^2 - a^2}}\, du = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C\ ;\ u^2 < a^2$$

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IMPROPER INTEGRAL

$$\int_a^{\infty} f(x)\,dx = \lim_{b\to\infty}\int_a^b f(x)\,dx$$

$$\int_{-\infty}^{b} f(x)\,dx = \lim_{a\to-\infty}\int_a^b f(x)\,dx$$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{0} f(x)\,dx + \int_0^{\infty} f(x)\,dx$$

$$= \lim_{a\to-\infty}\int_a^0 f(x)\,dx + \lim_{b\to\infty}\int_0^b f(x)\,dx$$

$$\int_a^b f(x)\,dx = \lim_{c\to b^-}\int_a^c f(x)\,dx$$

$$\int_a^b f(x)\,dx = \lim_{c\to a^+}\int_c^b f(x)\,dx$$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

$$= \lim_{m\to c^-}\int_a^m f(x)\,dx + \lim_{n\to c^+}\int_m^b f(x)\,dx$$

AREA OF REGION

$$A = \int_a^b \left[f(x) - g(x)\right] dx \qquad \text{OR} \qquad A = \int_c^d \left[w(y) - v(y)\right] dy$$

VOLUME OF REVOLUTION

$$V = \pi\int_a^b \left\{\left[f(x)\right]^2 - \left[g(x)\right]^2\right\} dx \qquad \textbf{OR} \qquad V = \pi\int_c^d \left\{\left[w(y)\right]^2 - \left[v(y)\right]^2\right\} dy$$

ARC LENGTH

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SIMPSON'S RULE

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[\left(f_0+f_n\right)+4\sum_{\substack{i=1\\ i\text{ odd}}}^{n-1} f_i+2\sum_{\substack{i=2\\ i\text{ even}}}^{n-2} f_i\right] \quad ; \quad n=\frac{b-a}{h};\ x_i=a+ih$$

TRAPEZOIDAL RULE

$$\int_a^b f(x)\,dx \approx \frac{h}{2}\left[\left(f_0+f_n\right)+2\sum_{i=1}^{n-1} f_i\right] \quad ; \quad n=\frac{b-a}{h};\ x_i=a+ih$$