UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2009/2010

SUBJECT	:	MATHEMATICS II
CODE	:	DSM 1933
COURSE	:	1 DEE/DET 2 DEE/DET
DATE	:	APRIL 2010
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER ALL QUESTIONS IN **PART A** AND **THREE (3)** QUESTIONS IN **PART B**

THIS EXAMINATION PAPER CONSISTS OF 8 PAGES

PART A

Q1 (a) Given $x = 3t^2 - 4$ and $y = t^3 + 5$, find $\dfrac{d^2y}{dx^2}$.

(4 marks)

(b) If $y = f(x)$, find $\dfrac{dy}{dx}$ for each of the equations below.

(i) $(2x + y)^2 = 3x + xy$.

(ii) $y = \ln(\sin x \cos x)$.

(9 marks)

(c) Evaluate $\dfrac{d^2y}{dx^2}$ at $x = \dfrac{\pi}{3}$ if $y = 3x^2 - 2\cos 3x$.

(3 marks)

(d) Find an equation of a tangent line that touches $f(x) = 2x^4 - \dfrac{1}{4}x^3 + 3x + 7$ at point (-2, 33).

(4 marks)

Q2 (a) Find the Laplace transforms for the functions below.

(i) $f(t) = 2t^3 - 5t + e^{-3t}$.

(ii) $f(t) = (\cos t - \sin t)^2$.

(iii) $g(t) = \begin{cases} 3, & 0 \le t < 2, \\ -1, & 2 \le t < 5, \\ 0, & t \ge 5. \end{cases}$

[*Use*: $g(t) = g_1 + [g_2 - g_1]H(t-a) + [g_3 - g_2]H(t-b)$]

(9 marks)

(b) Find the inverse Laplace transforms for the functions below.

(i) $F(s) = \dfrac{1}{s} + \dfrac{2}{s-3}$.

(ii) $F(s) = \dfrac{3}{(s+3)(s-3)}$.

(iii) $F(s) = \dfrac{7}{(s+6)^3}$.

(iv) $F(s) = \dfrac{4s+1}{s^2 - s - 2}$.

(11 marks)

PART B

Q3 (a) Given $A = \begin{pmatrix} 2 & 1 & -3 \\ 5 & -3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -3 \\ 4 & -2 \end{pmatrix}$ and $C = \begin{pmatrix} 2 & -1 \\ 3 & 0 \\ 5 & -1 \end{pmatrix}$. Find

(i) $A^{\mathrm{T}} - 3C$,

(ii) $AC + 2B$,

(iii) CB^{-1}.

(10 marks)

(b) Find the determinant of $H = \begin{pmatrix} 1 & 4 & -2 \\ 2 & -3 & 1 \\ 3 & 2 & -1 \end{pmatrix}$.

(3 marks)

(c) Solve the systems below using Gauss-Seidel iteration method with $x^{(0)} = y^{(0)} = z^{(0)} = 0$. Stop the iteration when the solution is accurate to three decimal places.

$$\begin{aligned} 4x + y &= 3.5, \\ 2y + 5z &= 1.5, \\ x + 3z &= 5.4. \end{aligned}$$

(7 marks)

Q4 (a)

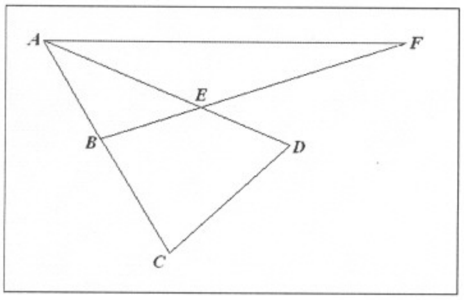

Figure Q4(a)

In **Figure Q4(a)**, given $AF = 5\mathbf{u}$, $AC = 4\mathbf{v}$, $\mathbf{BF} = 3\mathbf{BE}$, $2\mathbf{AD} = 3\mathbf{AE}$ and B is the midpoint of AC. Express $\mathbf{AB}$, $\mathbf{BF}$, $\mathbf{AD}$ and $\mathbf{CD}$ in terms of $\mathbf{u}$ and $\mathbf{v}$.

(7 marks)

(b) Given vectors $\mathbf{u} = 5\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ and $\mathbf{v} = \mathbf{i} - 3\mathbf{j} - 2\mathbf{k}$. Find

(i) $\mathbf{u} \cdot \mathbf{v}$,

(ii) $\mathbf{u} \times \mathbf{v}$,

(iii) the angle between vectors $\mathbf{u}$ and $\mathbf{v}$.

(7 marks)

(c) A line passes through two points $Q(4, 2, -1)$ and $R(2, 3, 5)$. Represent the line in

(i) parametric equation.

(ii) symmetric equation.

(3 marks)

(d) Given a plane $x - 2y + 3z = 4$. Find

(i) the distance between the point (2, 2, -1) and the plane.

(ii) a normal vector to the plane.

(3 marks)

Q5 (a) Simplify the following.

(i) i^{18}.

(ii) $3[\cos 35° + i\sin 35°] \times 7[\cos 215° - i\sin 215°]$.

Express the answer in standard form.

(4 marks)

(b) Given $z = 5 - 12i$. Find

(i) $3z - 4\bar{z}$, and write the answer in polar form.

(ii) $z\bar{z}$.

(6 marks)

(c) By using De Moivre's theorem,

(i) evaluate $(2-3i)^4$ and express the answer in standard form.

(ii) find three distinct cube roots of $2-3i$. Then, sketch three vectors of the respective roots on a single Argand plane.

(10 marks)

Q6 (a) Find the limits below.

(i) $\lim_{x\to 1} \dfrac{5x}{3x^2-2}$.

(ii) $\lim_{x\to 0} \dfrac{\sqrt{x+4}-2}{3x}$.

(iii) $\lim_{x\to 0} \left(\dfrac{\sin 5x}{2x}\right)$.

(iv) $\lim_{x\to 4} \sin\left(\dfrac{1}{2}-\dfrac{1}{x}\right)$.

(12 marks)

(b) Refer to **Figure Q6(b)** .

(i) Find the left-hand limits, the right-hand limits, and the limits of $y = f(x)$ as x approaching -2, 0 and 3 .

(ii) Is the function continuous at x = -2, 0 and 3?

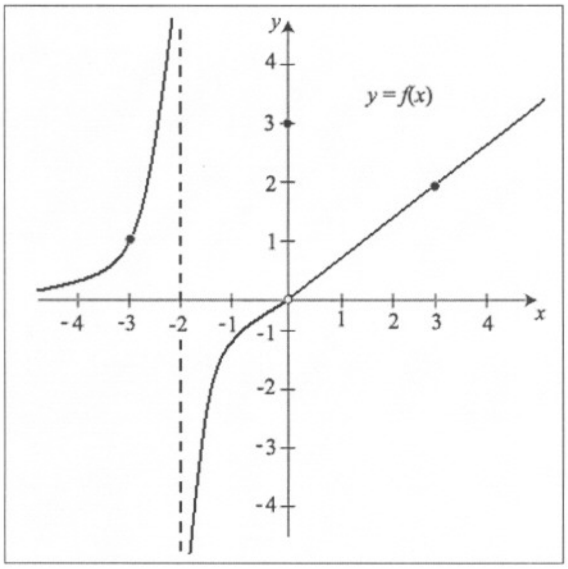

Figure Q6(b)

(8 marks)

FORMULAE

Table 1 : Laplace transform.

$L\{f(t)\}=\int_0^\infty f(t)e^{-st}\,dt=F(s)$	
$f(t)$	$F(s)$
k	$\dfrac{k}{s}$
$t^n,\ n = 1, 2, ..$	$\dfrac{n!}{s^{n+1}}$
e^{at}	$\dfrac{1}{s-a}$
$\sin at$	$\dfrac{a}{s^2+a^2}$
$\cos at$	$\dfrac{s}{s^2+a^2}$
$\sinh at$	$\dfrac{a}{s^2-a^2}$
$\cosh at$	$\dfrac{s}{s^2-a^2}$
$e^{at}f(t)$	$F(s-a)$
$t^n f(t),\ n = 1, 2, ..$	$(-1)^n \dfrac{d^n F(s)}{ds^n}$
$f(t-a)\,H(t-a)$	$e^{-as}F(s)$

Table 3 : Trigonometry Identities.

$\sin^2 x+\cos^2 x=1$
$\sin 2x=-2\sin x\cos x$
$\cos 2x=\cos^2 x-\sin^2 x$

Table 2: Differentiation
$\dfrac{d}{dx}x^n = nx^{n-1}$
$\dfrac{d}{dx}[f(x)g(x)] = f(x)g'(x)+g(x)f'(x)$
$\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{g(x)f'(x)-f(x)g'(x)}{(g(x))^2}$
$\dfrac{d^2y}{dx^2} = \dfrac{d}{dx}\left(\dfrac{dy}{dx}\right) = \dfrac{d}{dt}\left(\dfrac{dy}{dx}\right)\cdot\left(\dfrac{dt}{dx}\right)$
$\dfrac{d}{dx}\ln u = \dfrac{1}{u}\cdot\dfrac{du}{dx}$
$\dfrac{d}{dx}e^u = e^u\cdot\dfrac{du}{dx}$
$\dfrac{d}{dx}\sin u = \cos u\cdot\dfrac{du}{dx}$
$\dfrac{d}{dx}\tan u = \sec^2 u\cdot\dfrac{du}{dx}$
$\dfrac{d}{dx}\cot u = -\csc^2 u\cdot\dfrac{du}{dx}$
$\dfrac{d}{dx}\sec u = \sec u\tan u\cdot\dfrac{du}{dx}$
$\dfrac{d}{dx}\csc u = -\csc u\cot u\cdot\dfrac{du}{dx}$
$\dfrac{d}{dx}\cos u = -\sin u\cdot\dfrac{du}{dx}$