UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2009/2010

SUBJECT	:	MATHEMATICS III
CODE	:	DSM 2913
COURSE	:	1 DFA 2 DDM / DDT / DFA / DFT 3 DFA / DFX
DATE	:	APRIL 2010
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER ALL QUESTIONS IN **PART A** AND **FOUR (4)** QUESTIONS IN **PART B**

THIS EXAMINATION PAPER CONSISTS OF 7 PAGES

PART A

Q1 (a) Find the Laplace transform of the following functions.

(i) $f(t) = t^2 - e^{-9t} + 5 - \sinh\sqrt{2}t$.

(ii) $f(t) = \left(-t + 3e^{-4t}\right)\cos 5t$.

(12 marks)

(b) Consider the piecewise function

$$g(t) = \begin{cases} t & 0 < t \le 1, \\ 2 - t & t \ge 1. \end{cases}$$

(i) Express $g(t)$ in the form of Heaviside function.

(ii) Find the Laplace transform of $g(t)$.

(5 marks)

Q2 (a) Obtain the inverse Laplace transform for

$$\frac{s+1}{s^2 + 2s + 5}.$$

(4 marks)

(b) By using convolution theorem, evaluate

$$\mathcal{L}^{-1}\left\{\frac{1}{(s+1)(s-2)}\right\}.$$

(6 marks)

Q3 Consider the following initial-value problem

$$y'' + 9y = e^t, \qquad y(0) = 0, \qquad y'(0) = 0.$$

Determine the solution of the initial value problem by using Laplace transform.

(13 marks)

PART B

Q4 (a) Given matrices $A=\begin{pmatrix} -3 & -1 & -4 \\ 1 & 0 & 1 \\ 0 & -1 & 6 \end{pmatrix}$, $B=\begin{pmatrix} -1 & 1 & 0 \\ 0 & 2 & 1 \\ 5 & -3 & 0 \end{pmatrix}$ and $C=\begin{pmatrix} 2 & 3 & 0 \\ 1 & -2 & 1 \end{pmatrix}$.

Find

(i) $2A+3B$.

(ii) $|B|$.

(iii) BC^T.

(8 marks)

(b) Solve the system below by using the Gauss-Jordan elimination method.

$$x+2y-z=6,$$
$$3x+8y+9z=10,$$
$$2x-y+2z=-2.$$

(7 marks)

Q5 (a) Let $\mathbf{u}=4\mathbf{i}+\mathbf{k}$, $\mathbf{v}=2\mathbf{i}-\mathbf{j}$ and $\mathbf{w}=a\mathbf{i}+3\mathbf{j}-4b\mathbf{k}$. Find

(i) $4\mathbf{u}-3\mathbf{v}+\mathbf{w}$.

(ii) $\mathbf{u}\times\mathbf{v}$.

(iii) the value of a and b if $4\mathbf{u}-3\mathbf{v}+\mathbf{w}=\mathbf{u}\times\mathbf{v}$.

(9 marks)

(b) Find the distance from a plane with equation $2x-y-2z=4$ to point $(3,4,7)$.

(3 marks)

(c) Find the parametric and symmetric equations that pass through points $P(-2,0,3)$ and $Q(3,5,-2)$.

(3 marks)

Q6 (a) Given $z_1 = \dfrac{(2+i)(3+i)}{(1-2i)}$ and $z_2 = \dfrac{(2+3i)}{(1+2i)}$. Solve for $\overline{z_1 + z_2}$.

(6 marks)

(b) Let $z = \left[3\left(\cos 147^o + i\sin 147^o\right)\right]$. Find

(i) z^4.

(ii) $z^{\frac{1}{3}}$ and write your answer in $a + ib$ form, in three decimal places. Then, sketch it in an Argand diagram.

(9 marks)

Q7 (a) Solve the homogeneous differential equation

$$(xe^{\frac{y}{x}} + y)dx - xdy = 0.$$

(8 marks)

(b) Given

$$\frac{dy}{dx} + \frac{y}{2x} = \frac{1}{y}.$$

(i) Show that the differential equation above is an exact equation.
(ii) Then, solve the equation.

(7 marks)

Q8 Find the general solution of the second-order differential equation

$$y'' - 16y = 19.2e^{4x} + 60e^{x}.$$

(15 marks)

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Formulas

Laplace transform

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

$f(t)$	$F(s)$
k	$\frac{k}{s}$
$t^n,\ n = 1, 2, ..$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
$e^{at} f(t)$	$F(s-a)$
$t^n f(t)\ ,\ n = 1, 2, ..$	$(-1)^n \frac{d^n F(s)}{ds^n}$
$y(t)$	$Y(s)$
$y'(t)$	$sY(s) - y(0)$
$y''(t)$	$s^2Y(s) - sy(0) - y'(0)$
$f(t-a)H(t-a)$	$e^{-as} F(s)$
$f(t)\delta(t-a)$	$e^{-as} f(a)$
$\int_0^t f(\tau)\,d\tau$	$\frac{F(s)}{s}$

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Differentiation And Integration Formula

Differentiation	Integration
$\frac{d}{dx}x^n = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$
$\frac{d}{dx}\ln x = \frac{1}{x}$	$\int \frac{1}{x}\, dx = \ln\lvert x\rvert + C$
$\frac{d}{dx}e^x = e^x$	$\int e^x dx = e^x + C$
$\frac{d}{dx}\sin x = \cos x$	$\int \cos x\ dx = \sin x + C$
$\frac{d}{dx}\cos x = -\sin x$	$\int \sin x\ dx = -\cos x + C$
$\frac{d}{dx}\tan x = \sec^2 x$	$\int \sec^2 x\ dx = \tan x + C$
$\frac{d}{dx}\cot x = -\csc^2 x$	$\int \csc^2 x\ dx = -\cot x + C$
$\frac{d}{dx}\sec x = \sec x \tan x$	$\int \sec x \tan x\ dx = \sec x + C$
$\frac{d}{dx}\csc x = -\csc x \cot x$	$\int \csc x \cot x\ dx = -\csc x + C$
$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$
$\frac{d}{dx}\cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{1-x^2}} dx = \cos^{-1} x + C$
$\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$	$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$

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Characteristic Equation and General Solution

Differential equation : $ay'' + by' + cy = 0$;
Characteristic equation : $am^2 + bm + c = 0$

Case	Roots of the Characteristic Equation	General Solution
1	real and distinct : $m_1 \neq m_2$	$y = Ae^{m_1 x} + Be^{m_2 x}$
2	real and equal : $m_1 = m_2 = m$	$y = (A + Bx)e^{mx}$
3	imaginary : $m = \alpha \pm i\beta$	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$

$f(x)$	$y_p(x)$
$P_n(x) = A_0 + A_1 x + \cdots + A_n x^n$	$x^r(B_0 + B_1 x + \cdots + B_n x^n)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(p\cos\beta x + q\sin\beta x)$

Note : r is the least non-negative integer (r = 0, 1, or 2) which determine such that there is no terms in particular integral $y_p(x)$ corresponds to the complementary function $y_c(x)$.

Variation of Parameters Method for $ay'' + by' + cy = f(x)$

$$y(x) = uy_1 + vy_2$$

$$u = -\int \frac{y_2 f(x)}{W}\,dx \qquad v = \int \frac{y_1 f(x)}{W}\,dx$$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$