UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2009/2010

SUBJECT	:	ENGINEERING MATHEMATICS I
CODE	:	BSM 1913
COURSE	:	1 BDP / BEE / BFF 2 BDP / BEE / BFF 3 BDP / BEE / BFF 4 BDP / BEE
DATE	:	NOVEMBER 2009
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER ALL QUESTIONS IN **PART A** AND **THREE (3)** QUESTIONS IN **PART B**

THIS EXAMINATION PAPER CONSISTS OF 6 PAGES

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PART A

Q1 (a) If $y\sin^{-1}x = \sinh^{-1}y$, find $\dfrac{dy}{dx}$ by using implicit differentiation.

(5 marks)

(b) Find the arc length of the parametric curve $x = \cos^3 t$ and $y = \sin^3 t$ over the interval $t = \pi$ to $t = \dfrac{3}{2}\pi$.

(8 marks)

(c) Show that the area of surface that is generated by revolving the arc of $x^2 - 4y = 0$ about the y-axis from $y = 1$ to $y = 3$ is $\dfrac{4}{3}\pi(16 - 4\sqrt{2})$.

(7 marks)

Q2 (a) Determine the radius of convergence for the following power series

$$\sum_{n=0}^{\infty} \frac{(-3)^n}{n\,7^{n+1}}(x-5)^n .$$

(7 marks)

(b) Let $f(z) = e^z$. Obtain
(i) the Maclaurin series expansion of $f(x)$.
(ii) the Taylor series expansion of $f(x)$ at the point $z = 2$.

(7 marks)

(c) Given the power series of

$$\ln(1+x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1}.$$

Find the
(i) differentiation of the power series of $\ln(1+x)$.
(ii) integration of the power series of $\ln(1+x)$.

(6 marks)

PART B

Q3 (a) Evaluate

(i) $\lim_{x \to \infty} (\sqrt{x^2 + 4x} - x)$.

(ii) $\lim_{x \to \infty} 3xe^{-2x}$.

(iii) $\lim_{x \to 2} \frac{x^2 - 4}{e^{2-x} - 1}$.

(12 marks)

(b) Determine if the following function is continuous at $x = 0$.

$$f(x) = \begin{cases} \frac{x^2 - 9}{x - 3}, & -1 \le x < 0, \\ 3, & x = 0, \\ \frac{\sin x}{x} + 2, & 0 < x < 1. \end{cases}$$

(8 marks)

Q4 (a) If $x = \frac{1}{1 - t^2}$ and $y = \frac{1}{1 + t^2}$, find $\frac{dy}{dx}$ when $t = 2$.

(4 marks)

(b) **Figure Q4** below shows a window that consist a rectangle topped by a semicircle. The perimeter of the window is p. The area of the window is to be maximized.

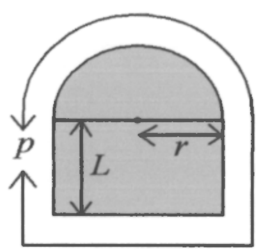

Figure Q4

(i) Assuming that the length of the rectangle is L and the radius of the semicircle is r. What is p in terms of L and r?

(ii) If $p = 12$, express the area of the window, A, in terms of r. Hence, find the radius of the semicircle, r, that maximizes the area, A.

(6 marks)

(c) Sketch the rational function

$$f(x) = \frac{1}{x^2 - 4}.$$

Show all the asymptote(s), intersection point(s), extremum and inflection point(s) (if any) in your sketch.

(10 marks)

Q5 (a) Evaluate $\int_0^3 (3-u)^2 e^{-4u}\, du$.

(5 marks)

(b) Use the substitution of $t = \tan\frac{x}{2}$ to calculate $\int_0^{\frac{\pi}{2}} \frac{dx}{3+5\cos x}$.

(8 marks)

(c) Find

(i) $\int_0^2 \frac{dx}{5+x^2}$.

(ii) $\int \frac{dx}{x\sqrt{9-16x^2}}$.

(7 marks)

Q6 (a) Discuss the convergence of the series $\sum_{n=0}^{\infty} \frac{3^n n!}{n^n}$.

(8 marks)

(b) Consider the power series

$$\sum_{n=0}^{\infty} \frac{1}{4^n}(x-1)^{2n+1}.$$

Find interval of convergence of the given power series.

(12 marks)

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Formulae

Indefinite Integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{1}{x}\, dx = \ln|x| + C$$

$$\int \cos x\, dx = \sin x + C$$

$$\int \sin x\, dx = -\cos x + C$$

$$\int \sec^2 x\, dx = \tan x + C$$

$$\int \csc^2 x\, dx = -\cot x + C$$

$$\int \sec x \tan x\, dx = \sec x + C$$

$$\int \csc x \cot x\, dx = -\csc x + C$$

$$\int e^x dx = e^x + C$$

$$\int \cosh x\, dx = \sinh x + C$$

$$\int \sinh x\, dx = \cosh x + C$$

$$\int \operatorname{sech}^2 x\, dx = \tanh x + C$$

$$\int \operatorname{csch}^2 x\, dx = -\coth x + C$$

$$\int \operatorname{sech} x \tanh x\, dx = -\operatorname{sech} x + C$$

$$\int \operatorname{csch} x \coth x\, dx = -\operatorname{csch} x + C$$

Integration of Inverse Functions

$$\int \frac{1}{\sqrt{1-x^2}}\, dx = \sin^{-1} x + C, \quad |x| < 1$$

$$\int \frac{-1}{\sqrt{1-x^2}}\, dx = \cos^{-1} x + C, \quad |x| < 1$$

$$\int \frac{1}{1+x^2}\, dx = \tan^{-1} x + C$$

$$\int \frac{-1}{1+x^2}\, dx = \cot^{-1} x + C$$

$$\int \frac{1}{|x|\sqrt{x^2-1}}\, dx = \sec^{-1} x + C, \quad |x| > 1$$

$$\int \frac{-1}{|x|\sqrt{x^2-1}}\, dx = \csc^{-1} x + C, \quad |x| > 1$$

$$\int \frac{1}{\sqrt{x^2+1}}\, dx = \sinh^{-1} x + C$$

$$\int \frac{1}{\sqrt{x^2-1}}\, dx = \cosh^{-1} x + C, \quad |x| > 1$$

$$\int \frac{-1}{|x|\sqrt{1-x^2}}\, dx = \operatorname{sech}^{-1} |x| + C, \quad 0 < x < 1$$

$$\int \frac{-1}{|x|\sqrt{1+x^2}}\, dx = \operatorname{csch}^{-1} |x| + C, \quad x \neq 0$$

$$\int \frac{1}{1-x^2}\, dx = \begin{cases} \tanh^{-1} x + C, & |x| < 1 \\ \coth^{-1} x + C, & |x| > 1 \end{cases}$$

TAYLOR AND MACLAURIN SERIES

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$

TRIGONOMETRIC SUBSTITUTION

Expression	***Trigonometry***	***Hyperbolic***
$\sqrt{x^2+k^2}$	$x = k\tan\theta$	$x = k\sinh\theta$
$\sqrt{x^2-k^2}$	$x = k\sec\theta$	$x = k\cosh\theta$
$\sqrt{k^2-x^2}$	$x = k\sin\theta$	$x = k\tanh\theta$

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Formulae

TRIGONOMETRIC SUBSTITUTION

$t=\tan\frac{1}{2}x$		$t=\tan x$	
$\sin x=\dfrac{2t}{1+t^2}$ $\tan x=\dfrac{2t}{1-t^2}$	$\cos x=\dfrac{1-t^2}{1+t^2}$ $dx=\dfrac{2dt}{1+t^2}$	$\sin 2x=\dfrac{2t}{1+t^2}$ $\tan 2x=\dfrac{2t}{1-t^2}$	$\cos 2x=\dfrac{1-t^2}{1+t^2}$ $dx=\dfrac{dt}{1+t^2}$

IDENTITIES OF TRIGONOMETRY AND HYPERBOLIC

Trigonometric Functions

$\cos^2 x+\sin^2 x=1$

$\sin 2x=2\sin x\cos x$

$\cos 2x=\cos^2 x-\sin^2 x$

$\quad=2\cos^2 x-1$

$\quad=1-2\sin^2 x$

$1+\tan^2 x=\sec^2 x$

$1+\cot^2 x=\csc^2 x$

$\tan 2x=\dfrac{2\tan x}{1-\tan^2 x}$

$\tan(x\pm y)=\dfrac{\tan x\pm\tan y}{1\mp\tan x\tan y}$

$\sin(x\pm y)=\sin x\cos y\pm\sin y\cos x$

$\cos(x\pm y)=\cos x\cos y\mp\sin x\sin y$

$2\sin ax\cos bx=\sin(a+b)x+\sin(a-b)x$

$2\sin ax\sin bx=\cos(a-b)x-\cos(a+b)x$

$2\cos ax\cos bx=\cos(a-b)x+\cos(a+b)x$

Hyperbolic Functions

$\sinh x=\dfrac{e^x-e^{-x}}{2}$

$\cosh x=\dfrac{e^x+e^{-x}}{2}$

$\cosh^2 x-\sinh^2 x=1$

$\sinh 2x=2\sinh x\cosh x$

$\cosh 2x=\cosh^2 x+\sinh^2 x$

$\quad=2\cosh^2 x-1$

$\quad=1+2\sinh^2 x$

$1-\tanh^2 x=\text{sech}^2 x$

$\coth^2 x-1=\text{csch}^2 x$

$\tanh 2x=\dfrac{2\tanh x}{1+\tanh^2 x}$

$\tanh(x\pm y)=\dfrac{\tanh x\pm\tanh y}{1\pm\tanh x\tanh y}$

$\sinh(x\pm y)=\sinh x\cosh y\pm\sinh y\cosh x$

$\cosh(x\pm y)=\cosh x\cosh y\pm\sinh x\sinh y$

CURVATURE, ARC LENGTH AND SURFACE AREA OF REVOLUTION

$$\kappa=\frac{\left|\dfrac{d^2y}{dx^2}\right|}{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}$$

$$\kappa=\frac{|\dot{x}\ddot{y}-\dot{y}\ddot{x}|}{\left[\dot{x}^2+\dot{y}^2\right]^{3/2}}$$

$$L=\int_{t_1}^{t_2}\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$$

$$L=\int_{x_1}^{x_2}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$L=\int_{y_1}^{y_2}\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

$$S=2\pi\int_{x_1}^{x_2}f(x)\sqrt{1+\left(\frac{d}{dx}[f(x)]\right)^2}\,dx$$

$$S=2\pi\int_{y_1}^{y_2}g(y)\sqrt{1+\left(\frac{d}{dy}[g(y)]\right)^2}\,dy$$