UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2009/2010

SUBJECT	:	ENGINEERING MATHEMATICS III
CODE	:	BSM 2913
COURSE	:	2 BFF/BEE/BDD
DATE	:	NOVEMBER 2009
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER ALL QUESTIONS IN **PART A** AND **THREE (3)** QUESTIONS IN **PART B**

THIS EXAMINATION PAPER CONSISTS OF 5 PAGES

PART A

Q1 (a) Evaluate the line integral

$$\int_C y \sin z \, ds$$

along the helix C that is represented by the parametric equations $x = \cos t$, $y = \sin t$, $z = t$ $(0 \le t \le 2\pi)$.

(10 marks)

(b) Use Green's Theorem to evaluate

$$\oint_C y^3 dx + (x^3 + 3xy^2) dy$$

where C is the circle $x^2 + y^2 = 9$ oriented counter clockwise.

(10 marks)

Q2 (a) Use Gauss's Theorem to find the flux of the vector field

$$\mathbf{F}(x, y, z) = x\,\mathbf{i} + 2y\,\mathbf{j} + z\,\mathbf{k}$$

across surfaces σ of the solid G enclosed by the planes $2x + 2y + z = 6$, $x = 0$, $y = 0$ and $z = 0$ oriented upward.

(10 marks)

(b) Use Stokes' Theorem to evaluate the work done by the force field

$$\mathbf{F}(x, y, z) = xz\,\mathbf{i} + xy\,\mathbf{j} + 3xz\,\mathbf{k}$$

along the curve C where C is the intersection between the cylinder $x^2 + y^2 = 1$ and the plane $y + z = 5$ oriented counterclockwise.

(10 marks)

PART B

Q3 (a) Find the local minimum and saddle point(s) of the function

$$f(x, y) = x^4 + y^4 - 4xy + 1.$$

(8 marks)

(b) Use the total differential dz to approximate the change in $z = x^2 y - 2xy - 6$ as (x, y) moves from the point $(0,3)$ to the point $(-0.07, 2.98)$.

(6 marks)

(c) Find the tangent plane of the surface $f(x, y) = \sqrt{4 - x^2 - 2y^2}$ at the point $(1, -1, 1)$.

(6 marks)

Q4 (a) Sketch the solid enclosed by the hemisphere $z=\sqrt{9-x^2-y^2}$, below by the plane $z=0$ and inside the cylinder $x^2+y^2=4$. Sketch its projection on the xy-plane. Find the moment of inertia of the solid about z-axis. Given that the density function is $\delta(x,y,z)=z$.

(10 marks)

(b) Find the volume of the solid bounded above by the sphere $x^2+y^2+z^2=4z$ and below by the cone $z=\sqrt{x^2+y^2}$ using triple integrals in spherical coordinates.

(10 marks)

Q5 (a) Suppose that a particle moves along a circular helix such that its position at time t is given by a vector valued function $\mathbf{r}(t)=\sin t\,\mathbf{i}+2t\,\mathbf{j}+\cos t\,\mathbf{k}$.

(i) Find the unit tangent vector $\mathbf{T}(t)$.

(ii) Find the unit normal vector $\mathbf{N}(t)$.

(iii) Sketch the graph of $\mathbf{r}(t)$, for $0<t<2\pi$.

(12 marks)

(b) Assume that the velocity of a moving particle is $\mathbf{v}(t)=(t+2)\mathbf{i}+t^2\mathbf{j}+e^{-t/3}\mathbf{k}$ and the position at $t=0$ is $\mathbf{r}(0)=4\mathbf{i}-3\mathbf{k}$. Find the particle position at $t=1$.

(8 marks)

Q6 (a) Use Green's Theorem to evaluate

$$\oint_C y^3dx+(x^3+3xy^2)\,dy,$$

where C consists of the line segment C_1 from $(2,0,0)$ to $(3,4,5)$ followed by the vertical line segment C_2 from $(3,4,5)$ to $(3,4,7)$.

(10 marks)

(b) Given a force field

$$\mathbf{F}(x,y)=(3x^2+6xy^2)\mathbf{i}+(6x^2y+4y^2)\mathbf{j}$$

acting on a particle moving along curve C from point $A(1,0)$ to point $B(0,1)$.

(i) Show that $\mathbf{F}$ is conservative.

(ii) Find a potential function $\phi(x,y)$.

(ii) Hence, find the work done by force field $\mathbf{F}$.

(10 marks)

FINAL EXAMINATION

SEMESTER / SESSION: SEM I / 2009/2010 COURSE : BFF/BEE/BDD

SUBJECT : ENGINEERING MATHEMATICS III CODE : BSM 2913

Formulae

Polar coordinate: $x = r\cos\theta$, $y = r\sin\theta$, $\theta = \tan^{-1}(y/x)$, and $\iint_R f(x,y)dA = \iint_R f(r,\theta)\, r\, dr\, d\theta$

Cylindrical coordinate: $x = r\cos\theta$, $y = r\sin\theta$, $z = z$, $\iiint_G f(x,y,z)dV = \iiint_G f(r,\theta,z)\, r\, dz\, dr\, d\theta$

Spherical coordinate: $x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$, $z = \rho\cos\phi$, $x^2 + y^2 + z^2 = \rho^2$, $0 < \theta < 2\pi$, $0 < \phi < \pi$, and

$$\iiint_G f(x,y,z)dV = \iiint_G f(\rho,\phi,\theta)\rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$$

Directional derivative: $D_{\mathbf{u}} f(x,y) = (f_x\mathbf{i} + f_y\mathbf{j})\cdot\mathbf{u}$

Let $\mathbf{F}(x,y,z) = M\,\mathbf{i} + N\,\mathbf{j} + P\,\mathbf{k}$ is vector field, then

the **divergence** of $\mathbf{F} = \nabla\cdot\mathbf{F} = \dfrac{\partial M}{\partial x} + \dfrac{\partial N}{\partial y} + \dfrac{\partial P}{\partial z}$

the **curl** of $\mathbf{F} = \nabla\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ M & N & P \end{vmatrix} = \left(\dfrac{\partial P}{\partial y} - \dfrac{\partial N}{\partial z}\right)\mathbf{i} - \left(\dfrac{\partial P}{\partial x} - \dfrac{\partial M}{\partial z}\right)\mathbf{j} + \left(\dfrac{\partial N}{\partial x} - \dfrac{\partial M}{\partial y}\right)\mathbf{k}$

Let C is a smooth curve given by $\mathbf{r}(t) = x(t)\,\mathbf{i} + y(t)\,\mathbf{j} + z(t)\,\mathbf{k}$, t is parameter, then

the unit tangent vector: $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

the unit normal vector: $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

the binormal vector: $\mathbf{B}(t) = \mathbf{T}(t)\times\mathbf{N}(t)$

the curvature: $\kappa = \dfrac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|} = \dfrac{\|\mathbf{r}'(t)\times\mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$

the radius of curvature: $\rho = 1/\kappa$

Green Theorem: $\oint_C M\,dx + N\,dy = \iint_R \left(\dfrac{\partial N}{\partial x} - \dfrac{\partial M}{\partial y}\right) dA$

Gauss Theorem: $\iint_S \mathbf{F}\bullet\mathbf{n}\,dS = \iiint_G \nabla\bullet\mathbf{F}\,dV$

Stokes' Theorem: $\oint_C \mathbf{F}\bullet d\mathbf{r} = \iint_S (\nabla\times\mathbf{F})\bullet\mathbf{n}\,dS$

Arc length

If $\mathbf{r}(t) = x(t)\,\mathbf{i} + y(t)\,\mathbf{j}$, $t\in[a,b]$, then the **arc length** $s = \int_a^b \|\mathbf{r}'(t)\|dt = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2}\,dt$

If $\mathbf{r}(t) = x(t)\,\mathbf{i} + y(t)\,\mathbf{j} + z(t)\,\mathbf{k}$, $t\in[a,b]$, then the **arc length** $s = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}\,dt$

FINAL EXAMINATION

SEMESTER / SESSION: SEM I / 2009/2010 COURSE : BFF/BEE/BDD

SUBJECT : ENGINEERING MATHEMATICS III CODE : BSM 2913

Tangent Plane

$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$

Extreme of two variable functions

$G(x,y) = f_{xx}(x,y) f_{yy}(x,y) - \left(f_{xy}(x,y)\right)^2$

Case1: If $G(a,b) > 0$ and $f_{xx}(x,y) < 0$ then f has local maximum at (a,b)

Case2: If $G(a,b) > 0$ and $f_{xx}(x,y) > 0$ then f has local minimum at (a,b)

Case3: If $G(a,b) < 0$ then f has a saddle point at (a,b)

Case4: If $G(a,b) = 0$ then no conclusion can be made.

In 2-D: Lamina

Mass: $m = \iint_R \delta(x,y)\,dA$, where $\delta(x,y)$ is a density of lamina.

Moment of mass: (i) about y-axis, $M_y = \iint_R x\delta(x,y)\,dA$, (ii) about x-axis, $M_x = \iint_R y\delta(x,y)\,dA$

Centre of mass, $(\bar{x}, \bar{y}) = \left(\dfrac{M_y}{m}, \dfrac{M_x}{m}\right)$

Moment inertia: (i) $I_y = \iint_R x^2\delta(x,y)\,dA$, (ii) $I_x = \iint_R y^2\delta(x,y)\,dA$, (iii) $I_o = \iint_R (x^2 + y^2)\delta(x,y)\,dA$

In 3-D: Solid

Mass, $m = \iiint_G \delta(x,y,z)\,dV$. If $\delta(x,y,z) = c$, c is a constant, then $m = \iiint_G dA$ is volume.

Moment of mass

(i) about yz-plane, $M_{yz} = \iiint_G x\delta(x,y,z)\,dV$

(ii) about xz-plane, $M_{xz} = \iiint_G y\delta(x,y,z)\,dV$

(iii) about xy-pane, $M_{xy} = \iiint z\delta(x,y,z)\,dV$

Centre of gravity, $(\bar{x}, \bar{y}, \bar{z}) = \left(\dfrac{M_{yz}}{m}, \dfrac{M_{xz}}{m}, \dfrac{M_{xy}}{m}\right)$

Moment inertia

(i) about x-axis: $I_x = \iiint_G (y^2 + z^2)\delta(x,y,z)\,dV$

(ii) about y-axis: $I_y = \iiint_G (x^2 + z^2)\delta(x,y,z)\,dV$

(iii) about z-axis: $I_z = \iiint_G (x^2 + y^2)\delta(x,y,z)\,dV$