UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2010/2011

COURSE NAME	:	ENGINEERING MATHEMATICS II E
SUBJECT CODE	:	BSM 1933 / BWM 10303
PROGRAM	:	1 BEF, BEU, BEE
EXAMINATION DATE	:	APRIL / MAY 2011
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS EXAMINATION PAPER CONSISTS OF SIX (6) PAGES

PART A

Q1 (a) Solve

(i) $(2x + 3\cos y)dx + (2y - 3x\sin y)dy = 0$.

(ii) $(1 + x^2)\dfrac{dy}{dx} - xy = x(1 + x^2), \quad y(0) = 3.$

(14 marks)

(b) Figure Q1 shows a RC circuit.

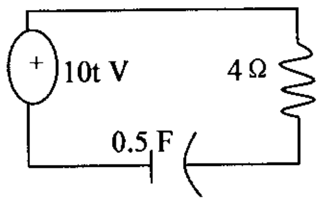

Figure Q1

By applying Kirchhoff's Law, show that the RC circuit can be modeled as

$$4\frac{di}{dt} + 2i = 10.$$

Hence, solve the governing equation by method of separation of variables.

(6 marks)

Q2 (a) Find the general solution of second-order differential equation by method of undetermined coefficients

$$\frac{d^2y}{dx^2} + 4y = \sin x \cos 2x,$$

which satisfies the conditions $y = 1$ and $\dfrac{dy}{dx} = 1$ when $x = 0$.

(10 marks)

(b) Find the general solution of the differential equation by variation of parameters

$$y'' - 4y' + 4y = (x + 1)e^{2x}.$$

(10 marks)

PART B

Q3 (a)

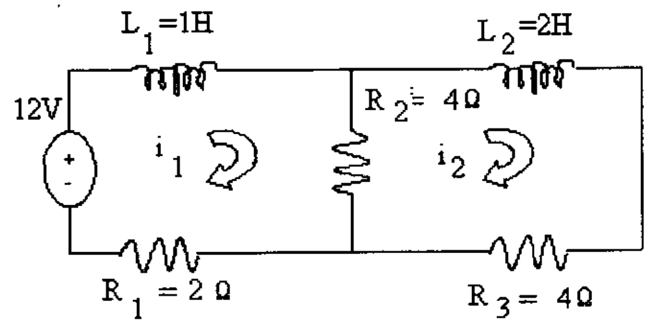

Figure Q3

Refer to the circuit network in figure Q3 above, show that a model for the current $i_1(t)$ and $i_2(t)$ is given by

$$\begin{pmatrix} i_1' \\ i_2' \end{pmatrix} = \begin{pmatrix} -6 & 4 \\ 2 & -4 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} + \begin{pmatrix} 12 \\ 0 \end{pmatrix}$$

Hence, find the general solution of homogeneous for $i_1(t)$ and $i_2(t)$ in above circuit.

(10 marks)

(b) Use a power series to solve the differential equation $y'' + xy' + y = 0$.

(10 marks)

Q4 (a) (i) Express $\dfrac{1}{(s+1)(s+2)^2}$ in partial fraction and show that

$$L^{-1}\left\{ \frac{1}{(s+1)(s+2)^2} \right\} = e^{-t} - (1+t)e^{-2t}.$$

(ii) Use the result in (i), to solve the inital value problem

$$y'' + 4y' + 4y = f(t), \qquad y(0) = 0 \text{ and } y'(0) = -1,$$

where

$$f(t) = \begin{cases} 0, & 0 \le t < 2, \\ e^{-(t-2)}, & t > 2. \end{cases}$$

(12 marks)

(b) Find the Laplace transform for

$$E(t) = \begin{cases} 1, & 0 \le t < 1 \\ 0, & 1 \le t < 2 \end{cases}$$

$$E(t) = E(t+2).$$

(8 marks)

Q5 (a) Find Fourier transform of the following function:

(a) $f(t) = e^{-at}$, where $0 < t < a$.

(b) $f(t) = t^3 \delta(2t - 3)$.

(c) $f(t) = e^{-at} \cos(\omega_0 t) H(t)$.

(10 marks)

(b) A periodic function $f(x)$ is defined as

$$f(x) = \begin{cases} 0, & -\pi < x < 0, \\ \sin x, & 0 \le x \le \pi, \end{cases}$$

and

$$f(x) = f(x + 2\pi).$$

(a) Sketch the graph of the function for $-3\pi < x < 3\pi$.

(b) Find the Fourier coefficients corresponding to the function.

(c) Write the corresponding Fourier series.

(d) By choosing appropriate value of x, deduce the sum of the infinite series $\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \frac{1}{7\cdot 9} + \cdots$.

(10 marks)

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Laplace Transforms

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
t^n, $n = 1, 2, 3, \ldots$	$\frac{n!}{s^{n+1}}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
e^{at}	$\frac{1}{s-a}$	$\delta(t-a)$	e^{-as}
$\sin at$	$\frac{a}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\cos at$	$\frac{s}{s^2+a^2}$	$\int_0^t f(u)g(t-u)du$	$F(s).G(s)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$y(t)$	$Y(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$\dot{y}(t)$	$sY(s)-y(0)$
$e^{at}f(t)$	$F(s-a)$	$\ddot{y}(t)$	$s^2Y(s)-sy(0)-\dot{y}(0)$
$t^n f(t)$, $n = 1, 2, 3, \ldots$	$(-1)^n \frac{d^n F(s)}{ds^n}$		

Periodic Function for Laplace transform: $\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)\,dt, \quad s > 0.$

Fourier Transform: $F\{f(t)\} = \int_{-\infty}^{\infty} e^{-iwt}f(t)dt$

Fourier Series: $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$, where $-\pi < x < \pi$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx;\ a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx \text{ and } b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx$$

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FIRST ORDER LINEAR DIFFERENTIAL EQUATION: $a\frac{dy}{dx}+by=f(x)$

Linear: $\frac{dy}{dx}+P(x)y=Q(x)$ and I. F = $e^{\int P(x)dx}$; $y\, e^{\int P(x)dx} = \int e^{\int P(x)dx}Q(x)\,dx+C$

Exact: $M(x,y)dx+N(x,y)dy=0$ and $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$

SECOND ORDER LINEAR DIFFERENTIAL EQUATION: $a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=f(x)$

Homogeneous Solution:

Type of roots	Complementary Function
Roots are real and distinct $(m_1 \neq m_2)$	$y_c = Ae^{m_1x}+Be^{m_2x}$
Roots are real and equal $(m_1 = m_2)$	$y_c = (A+xB)e^{mx}$
Roots are imaginary $(m=\alpha \pm i\beta)$	$y_c = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Method of undetermined coefficients:

$f(x)$	$y_p(x)$
$P_n(x)e^{\lambda x}$	$x^r(B_nx^n+B_{n-1}x^{n-1}+\cdots+B_1x\ +B_0)\ e^{\lambda x}$
$P_n(x)\begin{cases}\cos\omega x\\ \sin\omega x\end{cases}$	$x^r(B_nx^n+B_{n-1}x^{n-1}+\cdots+B_1x\ +B_0)\cos\omega x$ $x^r(C_nx^n+C_{n-1}x^{n-1}+\cdots+C_1x\ +C_0)\sin\omega x$
$Ce^{\lambda x}\begin{cases}\cos\omega x\\ \sin\omega x\end{cases}$	$x^r\ e^{\lambda x}\ K\cos\omega x+L\sin\omega x)$
$P_n(x)e^{\lambda x}\begin{cases}\cos\omega x\\ \sin\omega x\end{cases}$	$x^r(B_nx^n+B_{n-1}x^{n-1}+\cdots+B_1x\ +B_0)e^{\lambda x}\cos\omega x$ $x^r(C_nx^n+C_{n-1}x^{n-1}+\cdots+C_1x\ +C_0)e^{\lambda x}\sin\omega x$

Variation of Parameters:

$y_c = y_1 + y_2$; $W=\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$; $u=-\int\frac{y_2 f(x)}{aW}dx+A$ and $v=\int\frac{y_1 f(x)}{aW}dx+B$

General Solution $y=uy_1+vy_2$

SYSTEM OF FIRST ORDER LINEAR DIFFERENTIAL EQUATION

Eigen value and Eigen vector: $|A-\lambda I|=0$.