UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2010/2011

COURSE NAME	:	ENGINEERING MATHEMATICS III
COURSE CODE	:	BSM 2913/BWM 20403
PROGRAMME	:	2 BDD/BEE/BFF
EXAMINATION DATE	:	APRIL/MAY 2011
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS EXAMINATION PAPER CONSISTS OF FIVE (5) PAGES

Q1 (a) Find the local extremums and saddle point (if exists) for the function

$$f(x,y) = 2x^4 + y^2 - 2xy.$$

(10 marks)

(b) Use the spherical coordinates to evaluate

$$\int_0^{\sqrt{2}} \int_{-\sqrt{2-y^2}}^{\sqrt{2-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} \left(x^2 + y^2 + z^2\right)^{\frac{3}{2}} dz\, dx\, dy.$$

(10 marks)

Q2 (a) The displacement of a particle at time t is given by

$$\mathbf{r}(t) = 3\cos t\,\mathbf{i} + 3\sin t\,\mathbf{j} + 4t\,\mathbf{k},\ 0 \le t \le 2\pi.$$

(i) Find the velocity $\mathbf{v}(t)$, the unit tangent vector $\mathbf{T}(t)$, and the unit normal vector $\mathbf{N}(t)$.

(ii) Sketch the graph of $\mathbf{r}(t)$.

(10 marks)

(b) Find the moment of inertia about z-axis, I_z of a solid G, where G is the solid in the first octant bounded by paraboloids

$$z = x^2 + y^2 \text{ and } z = 2 - x^2 - y^2$$

with a density function given by

$$\delta(x,y,z) = \sqrt{x^2 + y^2}.$$

(10 marks)

Q3 (a) Evaluate the line integral

$$\int_C (x^2 + y^2 - x)\,dx + y\sqrt{x^2 + y^2}\,dy$$

where C is the path consists of a line segment from $(0,0)$ to $(1,0)$ and along a semicircle $y = \sqrt{1-x^2}$ from $(1,0)$ to $(-1,0)$.

(10 marks)

(b) Given the vector field

$$\mathbf{F}(x,y,z) = (2xy + z^3)\,\mathbf{i} - x^2\,\mathbf{j} + 3xz^2\,\mathbf{k}$$

(i) Show that $\mathbf{F}$ is a conservative vector field.

(ii) Find its potential function ϕ.

(iii) Hence, find the work done in this field to move an object from a point $(1,1,1)$ to $(2,2,2)$.

(10 marks)

Q4 (a) Use Green's theorem to evaluate the integral

$$\oint_C \left(e^x + y^2\right)dx + \left(e^y + x^2\right)dy$$

where C is the boundary of the region enclosed by $y = x^2$ and $x = y^2$, and curve C is oriented counterclockwise.

(10 marks)

(b) Use Stokes' theorem to evaluate the work done by the force field

$$\mathbf{F}(x,y,z) = xz\,\mathbf{i} + xy\,\mathbf{j} + 3xz\,\mathbf{k}$$

along the curve C where C is the intersection between the cylinder $x^2 + y^2 = 4$ and the plane $y + z = 3$ oriented counter clockwise.

(10 marks)

Q5 (a) Let σ be surface of $z = 5 - x^2 - y^2$ for $z \geq 1$ oriented upward. The vector field $\mathbf{F}(x,y,z) = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$ across surface σ. Find the flux of vector field $\mathbf{F}$, $\iint_\sigma \mathbf{F} \cdot \mathbf{n}\, dS$.

(10 marks)

(b) Use the Gauss's Theorem to evaluate the flux of vector field $\mathbf{F}$ across surface σ, where $\mathbf{F}(x,y,z) = xy^2\,\mathbf{i} + yz^2\,\mathbf{j} + x^2z\,\mathbf{k}$ and σ is the surface of the solid bounded above by sphere $x^2 + y^2 + z^2 = 4$ and below by the cone $z = \sqrt{x^2 + y^2}$.

(10 marks)

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Formulae

Polar coordinate: $x = r\cos\theta$, $y = r\sin\theta$, $\theta = \tan^{-1}(y/x)$, and $\iint_R f(x,y)dA = \iint_R f(r,\theta)\, r\, dr\, d\theta$

Cylindrical coordinate: $x = r\cos\theta$, $y = r\sin\theta$, $z = z$, $\iiint_G f(x,y,z)dV = \iiint_G f(r,\theta,z)\, r\, dz\, dr\, d\theta$

Spherical coordinate: $x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$, $z = \rho\cos\phi$, $x^2 + y^2 + z^2 = \rho^2$, $0 \le \theta \le 2\pi$, $0 \le \phi \le \pi$, and

$$\iiint_G f(x,y,z)dV = \iiint_G f(\rho,\phi,\theta)\rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$$

Directional derivative: $D_{\mathbf{u}} f(x,y) = (f_x \mathbf{i} + f_y \mathbf{j}) \cdot \mathbf{u}$

Let $\mathbf{F}(x,y,z) = M\,\mathbf{i} + N\,\mathbf{j} + P\,\mathbf{k}$ is vector field, then

the **divergence** of $\mathbf{F} = \nabla \cdot \mathbf{F} = \dfrac{\partial M}{\partial x} + \dfrac{\partial N}{\partial y} + \dfrac{\partial P}{\partial z}$

the **curl** of $\mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ M & N & P \end{vmatrix} = \left(\dfrac{\partial P}{\partial y} - \dfrac{\partial N}{\partial z}\right)\mathbf{i} - \left(\dfrac{\partial P}{\partial x} - \dfrac{\partial M}{\partial z}\right)\mathbf{j} + \left(\dfrac{\partial N}{\partial x} - \dfrac{\partial M}{\partial y}\right)\mathbf{k}$

Let C is a smooth curve given by $\mathbf{r}(t) = x(t)\,\mathbf{i} + y(t)\,\mathbf{j} + z(t)\,\mathbf{k}$, t is parameter, then

the unit tangent vector: $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

the unit normal vector: $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

the binormal vector: $\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$

the curvature: $\kappa = \dfrac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|} = \dfrac{\|\mathbf{r}'(t) \times \mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$

the radius of curvature: $\rho = 1/\kappa$

Green Theorem: $\oint_C M\,dx + N\,dy = \iint_R \left(\dfrac{\partial N}{\partial x} - \dfrac{\partial M}{\partial y}\right) dA$

Gauss Theorem: $\iint_S \mathbf{F} \bullet \mathbf{n}\, dS = \iiint_G \nabla \bullet \mathbf{F}\, dV$

Stokes' Theorem: $\oint_C \mathbf{F} \bullet d\mathbf{r} = \iint_S (\nabla \times \mathbf{F}) \bullet \mathbf{n}\, dS$

Arc length

If $\mathbf{r}(t) = x(t)\,\mathbf{i} + y(t)\,\mathbf{j}$, $t \in [a,b]$, then the **arc length** $s = \int_a^b \|\mathbf{r}'(t)\| dt = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2}\, dt$

If $\mathbf{r}(t) = x(t)\,\mathbf{i} + y(t)\,\mathbf{j} + z(t)\,\mathbf{k}$, $t \in [a,b]$, then the **arc length** $s = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}\, dt$

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Tangent Plane

$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$

Extreme of two variable functions

$G(x,y) = f_{xx}(x,y) f_{yy}(x,y) - \left(f_{xy}(x,y)\right)^2$

Case1: If $G(a,b) > 0$ and $f_{xx}(x,y) < 0$ then f has local maximum at (a,b)

Case2: If $G(a,b) > 0$ and $f_{xx}(x,y) > 0$ then f has local minimum at (a,b)

Case3: If $G(a,b) < 0$ then f has a saddle point at (a,b)

Case4: If $G(a,b) = 0$ then no conclusion can be made.

In 2-D: Lamina

Mass: $m = \iint_R \delta(x,y)\,dA$, where $\delta(x,y)$ is a density of lamina.

Moment of mass: (i) about y-axis, $M_y = \iint_R x\delta(x,y)\,dA$, (ii) about x-axis, $M_x = \iint_R y\delta(x,y)\,dA$

Centre of mass, $(\bar{x}, \bar{y}) = \left(\dfrac{M_y}{m}, \dfrac{M_x}{m}\right)$

Moment inertia: (i) $I_y = \iint_R x^2 \delta(x,y)\,dA$, (ii) $I_x = \iint_R y^2 \delta(x,y)\,dA$, (iii) $I_o = \iint_R \left(x^2 + y^2\right)\delta(x,y)\,dA$

In 3-D: Solid

Mass, $m = \iiint_G \delta(x,y,z)\,dV$. If $\delta(x,y,z) = c$, c is a constant, then $m = \iiint_G dA$ is volume.

Moment of mass

(i) about yz-plane, $M_{yz} = \iiint_G x\delta(x,y,z)\,dV$

(ii) about xz-plane, $M_{xz} = \iiint_G y\delta(x,y,z)\,dV$

(iii) about xy-pane, $M_{xy} = \iiint z\delta(x,y,z)\,dV$

Centre of gravity, $(\bar{x}, \bar{y}, \bar{z}) = \left(\dfrac{M_{yz}}{m}, \dfrac{M_{xz}}{m}, \dfrac{M_{xy}}{m}\right)$

Moment inertia

(i) about x-axis: $I_x = \iiint_G \left(y^2 + z^2\right)\delta(x,y,z)\,dV$

(ii) about y-axis: $I_y = \iiint_G \left(x^2 + z^2\right)\delta(x,y,z)\,dV$

(iii) about z-axis: $I_z = \iiint_G \left(x^2 + y^2\right)\delta(x,y,z)\,dV$