UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2010/2011

COURSE NAME	:	ENGINEERING MATHEMATICS IV
COURSE CODE	:	BWM 30603/BSM 3913
PROGRAMME	:	2 BDD/BEE/BFF 3 BDD/BEE/BFF 3 BDD/BEE/BFF
EXAMINATION DATE	:	APRIL/MAY 2011
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER ALL QUESTIONS IN **PART A** AND **TWO (2)** QUESTIONS IN **PART B.** ALL CALCULATIONS AND ANSWERS MUST BE IN THREE (3) DECIMAL PLACES.

THIS EXAMINATION PAPER CONSISTS OF 10 PAGES

PART A

Q1 (a) The temperature distribution $u(x,t)$ of one dimensional silver rod is governed by the heat equation

$$\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$$

with α^2 is thermal diffusity $=1.71$.

Given the initial condition,

$$u(x,0)=\begin{cases} x, & 0\le x\le 2,\\ 4-x, & 2\le x\le 4.\end{cases}$$

and boundary conditions,

$$u(0\,,t)=t,\quad u(4,t)=t^2.$$

Find the temperature distribution of the rod with $\Delta x=h=1$ and $\Delta t=k=0.2$ for $0\le t\le 0.4$ by using implicit Crank-Nicolson method.

(15 marks)

(b) The steady state temperature distribution $T(x,y)$ of a thin plate over the rectangle $0\le x\le 1,\ 0\le y\le 2$, satisfies the Laplace equation

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=0,\ 0\le x\le 1,\ 0\le y\le 2$$

with the boundary conditions,

$$T(0,y)=1,\ T(1,y)=e^y,\ 0\le y\le 2,$$
$$T(x,0)=1,\ T(x,2)=e^{2x},\ 0\le x\le 1.$$

By using finite-difference method with $h=\Delta x=k=\Delta y=0.5$, find the temperature distribution, $T(x,y)$ of the thin plate.

(10 marks)

Q2

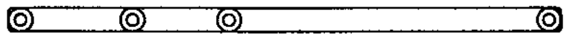

Figure Q2

Consider a fin of length 5 unit has four nodes and three elements, as shown in **Figure Q2**. The heat flow equation is given by

$$\frac{d}{dx}\left(A(x)k(x)\frac{dT(x)}{dx} \right) + Q(x) = 0, \text{ for } 0 \le x \le 5$$

with $A(x)$ is the cross-sectional area, $k(x)$ is the thermal conductivity, $T(x)$ is the temperature at length x and $Q(x)$ is the heat supply per unit time and per unit length.

Find the temperature at each nodal point, T_2, T_3 and T_4, if $A(x)$ is 30 unit, $k(x)$ is 10 unit and $Q(x)$ is 10 unit. Let the temperature at $x = 0$ is 0 unit and the heat flux, $-k\left.\frac{dT}{dx}\right|_{x=5} = 10$ unit.

(25 marks)

PART B

Q3 (a) Given $f(x)=\cosh\left(x-\frac{\pi}{2}\right)$ and $g(x)=3\sin(3x)$.

(i) Find the interval of the roots of $f(x)=g(x)$ from the following intervals; $[0.9\ ,\ 1.0]$, $[2.3\ ,\ 2.4]$ and $[2.8\ ,\ 2.9]$.

(ii) Hence, find the **most positive root** of $f(x)=g(x)$ by using **Bisection method** (iterate until $|f(c_i)|<\varepsilon$) **and Newton-Raphson method** (iterate until $|f(x_i)|<\varepsilon$).

(iii) If the exact solution of $f(x)=g(x)$ is 2.896, find the percentage of relative error for both methods.

(15 marks)

(b) A biologist has placed three strains of bacteria (denoted I, II and III) in a test tube, where they will feed on three different food sources (A, B and C). Each day 700 units of A, 400 units of B and 500 units of C are placed in the test tube. Each bacteria consumes a certain number of units of each food per day, as shown in **Figure Q3** below.

	Bacteria Strain I	Bacteria Strain II	Bacteria Strain III
Food A	0	1	2
Food B	5	1	0
Food C	1	3	1

Figure Q3

(i) Form a system of linear equations based on the above problem.

(ii) Hence, determine the number of bacteria of each strain that can coexist in the test tube and consume all of the food by using **Thomas Algorithm method.**

(10 marks)

Q4 (a) Construct natural cubic spline $S(x)$ using the following data given by **Table Q4(a)**.

x_i	-2	-1	0
$f(x_i)$	1.4	2.9	2.2

Table Q4(a)

(12 marks)

(b) A point P is moving along the curve whose equation is $y=\sqrt{x^3+17}$. By using 3-point central and 5-point difference formula with $h=0.05$, calculate how fast is P moving when $x=3.35$?

(6 marks)

(c) A basketball player makes a successful shot from the free throw line. Suppose that the path of the ball from the moment of release to the moment it enters the hoop is described by

$$y=2.15+2.09x-0.41x^2, \quad 0\le x\le 3.6$$

where x is the horizontal distance (in meters) from the point of release, and y is the vertical distance (in meters) above the floor. Approximate the distance of the ball travels from the moment of release to the moment it enters the hoop, by using the appropriate Simpson's rule with $h=0.4$.

[Hint: Arc length of the curve, $L=\int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$]

(7 marks)

Q5 (a) Given

$$A = \begin{pmatrix} 4 & 3 & 0 \\ 3 & 4 & -1 \\ 0 & -1 & 4 \end{pmatrix}.$$

By taking $v^{(0)} = (1 \quad 1 \quad 0)^T$, calculate the largest eigenvalue and its eigenvector by using power method.

(7 marks)

(b) The initial-value problem $y' = \frac{2y}{x} - xy^2, \quad y(1) = 5,$ has a unique solution $y(x) = \frac{20x^2}{5x^4 - 1}$. Approximate the solution at $x = 1.4$ using the fourth order Runge-Kutta method (RK4) with the same step size $h = 0.2$ and estimate the absolute error.

(8 marks)

(b) Solve the boundary-value problem, $y'' + xy = x^3 - \frac{4}{x}, \quad 1 \le x \le 2,$ with boundary conditions, $4y(1) + y'(1) = 0,$ and $3y(2) + 2y'(2) = 0.$ By using $h = \Delta x = 0.2$, derive the system of linear equations in matrix-vector form by finite-difference method **(do not solve the system)**.

(10 marks)

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FORMULAS

Nonlinear equations

Bisection : $c_i = \dfrac{a_i + b_i}{2}$, $i = 0,1,2,...$

Newton-Raphson method : $x_{i+1} = x_i - \dfrac{f(x_i)}{f'(x_i)}$, $i = 0,1,2,\ldots$

System of linear equations

Thomas algorithm:

i	**1**	**2**	**...**	***n***
d_i				
e_i				
c_i				
b_i				
$\alpha_1 = d_1$ $\alpha_i = d_i - c_i\beta_{i-1}$				
$\beta_i = \dfrac{e_i}{\alpha_i}$				
$y_1 = \dfrac{b_1}{\alpha_1}$ $y_i = \dfrac{b_i - c_i y_{i-1}}{\alpha_i}$				
$x_n = y_n$ $x_i = y_i - \beta_i x_{i+1}$				

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FORMULAS

Interpolation

Cubic spline:

$$S_k(x) = \frac{m_k}{6h_k}(x_{k+1}-x)^3 + \frac{m_{k+1}}{6h_k}(x-x_k)^3 + \left(\frac{f_k}{h_k} - \frac{m_k}{6}h_k\right)(x_{k+1}-x) + \left(\frac{f_{k+1}}{h_k} - \frac{m_{k+1}}{6}h_k\right)(x-x_k)$$

where $k = 0,1,2,\ldots,n-1$

$$\left.\begin{aligned} h_k &= x_{k+1} - x_k \\ d_k &= \frac{f_{k+1} - f_k}{h_k} \end{aligned}\right\}, \quad k = 0,1,2,\ldots,n-1$$

$$b_k = 6(d_{k+1} - d_k), \quad k = 0,1,2,\ldots,n-2$$

$$m_0 = 0$$
$$m_n = 0$$

$$h_k m_k + 2(h_k + h_{k+1})m_{k+1} + h_{k+1}m_{k+2} = b_k, \quad k = 0,1,2,\ldots,n-2$$

Numerical differentiation and integration

Differentiation:

First derivatives:

3-point central difference: $f'(x) \approx \dfrac{f(x+h) - f(x-h)}{2h}$

5-point difference: $f'(x) \approx \dfrac{-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)}{12h}$

Second derivatives:

3-point central difference: $f''(x) \approx \dfrac{f(x+h) - 2f(x) + f(x-h)}{h^2}$

5-point difference: $f''(x) \approx \dfrac{-f(x+2h) + 16f(x+h) - 30f(x) + 16f(x-h) - f(x-2h)}{12h^2}$

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FORMULAS

Integration:

Simpson's $\frac{1}{3}$ rule: $\int_a^b f(x)dx \approx \frac{h}{3}\left[f_0 + f_n + 4\sum_{\substack{i=1 \\ i\,\text{odd}}}^{n-1} f_i + 2\sum_{\substack{i=2 \\ i\,\text{even}}}^{n-2} f_i \right]$

Simpson's $\frac{3}{8}$ rule:

$$\int_a^b f(x)dx \approx \frac{3}{8}h\left[f_0 + f_n + 3(f_1 + f_2 + f_4 + f_5 + \ldots + f_{n-2} + f_{n-1}) + 2(f_3 + f_6 + \ldots + f_{n-6} + f_{n-3}) \right]$$

Eigenvalue

Power Method: $v^{(k+1)} = \frac{1}{m_{k+1}} A v^{(k)}, \qquad k = 0,1,2,.....$

Ordinary differential equations

Initial value problems:

Fourth-order Runge-Kutta Method: $y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

where $k_1 = hf(x_i, y_i)$ $\qquad k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2})$

$k_3 = hf(x_i + \frac{h}{2}, y_i + \frac{k_2}{2})$ $\qquad k_4 = hf(x_i + h, y_i + k_3)$

Boundary value problems:

Finite difference method:

$y_i' \approx \frac{y_{i+1} - y_{i-1}}{2h}$ $\qquad y_i'' \approx \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$

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FORMULAS

Partial differential equations

Heat equation- Implicit Crank-Nicolson:

$$\left(\frac{\partial u}{\partial t}\right)_{i,j+\frac{1}{2}} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j+\frac{1}{2}} \qquad \frac{u_{i,j+1} - u_{i,j}}{k} = \frac{c^2}{2}\left(\frac{u_{i+1,j+1} - 2u_{i,j+1} + u_{i-1,j+1}}{h^2} + \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2}\right)$$

Laplace Equation: Finite Difference Method

$$\left(\frac{\partial^2 u}{\partial x^2}\right)_{i,j} + \left(\frac{\partial^2 u}{\partial y^2}\right)_{i,j} = 0 \qquad \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{k^2} = 0$$

Finite element method

$$KT = F_b - F_l$$

where $K_{ij} = \int_p^q A(x)k(x)\frac{dN_i}{dx}\frac{dN_j}{dx}dx$ is stiffness matrix,

$$T = T_i$$

$$F_b = \left[N_i A(x)k(x)\frac{dT}{dx}\right]_p^q$$

$$F_l = -\int_p^q N_i Q(x)dx$$