CONFIDENTIAL

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2011/2012

COURSE NAME	:	ENGINEERING MATHEMATICS I
COURSE CODE	:	BSM 1913
PROGRAMME	:	4 BEE
EXAMINATION DATE	:	JUNE 2012
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL FIVE (5)** QUESTIONS

THIS EXAMINATION PAPER CONSISTS OF SIX (6) PAGES

CONFIDENTIAL

Q1 (a) Use the L'Hôpital's Rule to evaluate the following limits.

(i) $\lim\limits_{x\to+\infty} x^2e^{-3x}$.

(ii) $\lim\limits_{x\to 0} \dfrac{\tan x - x}{x-\sin x}$.

(iii) $\lim\limits_{x\to 1} \dfrac{\sin\left(1-\sqrt{x}\right)}{x-1}$.

(12 marks)

(b) Given the function

$$f(x)=\begin{cases} 2x^3+x+7, & x\le -1, \\ m(x+1)+k, & -1<x\le 2, \\ x^2+5, & x>2. \end{cases}$$

Determine the value of m and k such that the function $f(x)$ continuous everywhere.

(8 marks)

Q2 (a) Find $\dfrac{dy}{dx}$ of

(i) $y^2+x^3-xy+\cos y=0$ using implicit differentiation.

(ii) $x=\dfrac{1}{1-t^2}$ and $y=\dfrac{1}{1+t^2}$ when $t=2$ using parametric differentiation.

(iii) $y=\sin^2(2x)+\sqrt{x}$

(12 marks)

(b) The perimeter of a rectangular football field is 100 m. Find the maximum area of the field.

(8 marks)

Q3 (a) Evaluate

(i) $\int_0^1 x^3 e^{2x}\, dx$. [Use tabular method]

(ii) $\int \frac{\ln x}{x}\, dx$. [Use part by part method]

(iii) $\int \frac{\sqrt{3+\sqrt{x}}}{\sqrt{x}}\, dx$. [Use substitution method using $u = 3+\sqrt{x}$]

(14 marks)

(b) Find the area of the surface that is generated by revolving the portion of the curve $f(x)=\sqrt{4-x^2}$, $-1 \le x \le 1$, $360°$ about the x-axis

(6 marks)

Q4 (a) Find $\frac{dy}{dx}$ of

(i) $y\sin^{-1} x = \sinh^{-1} y$ using implicit differentiation.

(ii) $y(x) = \cot^{-1}(\sin x)$

(iii) $y(x) = \left(\sin^{-1} x\right)^3$

(12 marks)

(b) Evaluate the following integral.

(i) $\int \left(\text{sec h}^2 x\right)\sqrt{\tanh x}\, dx.$

(ii) $\int_0^{\sqrt{3}} \frac{1}{\sqrt{x^2+3}}\, dx.$

(8 marks)

Q5 (a) Find the Maclaurin series of $f(x) = \sin x$ up to three nonzero terms.

(7 marks)

(b) Use the result in **Q5(a)** to approximate $\int_0^1 \frac{\sin x}{x}\, dx.$

Write your answer in four decimal places.

(6 marks)

(c) Given that $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$.

(i) Derive the power series of $\sin x$.

(ii) Find the first three nonzero terms of the power series $\frac{\sin x}{\sqrt{x}}$.

(7 marks)

FINAL EXAMINATION

SEMESTER / SESSION : SEM II / 2011/2012
COURSE : ENGINEERING MATHEMATICS I

COURSE : 4 BEE
COURSE CODE : BSM 1913

Formulae

Indefinite Integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{1}{x}\, dx = \ln|x| + C$$

$$\int \cos x\, dx = \sin x + C$$

$$\int \sin x\, dx = -\cos x + C$$

$$\int \sec^2 x\, dx = \tan x + C$$

$$\int \csc^2 x\, dx = -\cot x + C$$

$$\int \sec x \tan x\, dx = \sec x + C$$

$$\int \csc x \cot x\, dx = -\csc x + C$$

$$\int e^x dx = e^x + C$$

$$\int \cosh x\, dx = \sinh x + C$$

$$\int \sinh x\, dx = \cosh x + C$$

$$\int \text{sech}^2 x\, dx = \tanh x + C$$

$$\int \text{csch}^2 x\, dx = -\coth x + C$$

$$\int \text{sech}\, x \tanh x\, dx = -\text{sech}\, x + C$$

$$\int \text{csch}\, x \coth x\, dx = -\text{csch}\, x + C$$

Integration of Inverse Functions

$$\int \frac{1}{\sqrt{1-x^2}}\, dx = \sin^{-1} x + C, \ |x| < 1$$

$$\int \frac{-1}{\sqrt{1-x^2}}\, dx = \cos^{-1} x + C, \ |x| < 1$$

$$\int \frac{1}{1+x^2}\, dx = \tan^{-1} x + C$$

$$\int \frac{-1}{1+x^2}\, dx = \cot^{-1} x + C$$

$$\int \frac{1}{|x|\sqrt{x^2-1}}\, dx = \sec^{-1} x + C, \ |x| > 1$$

$$\int \frac{-1}{|x|\sqrt{x^2-1}}\, dx = \csc^{-1} x + C, \ |x| > 1$$

$$\int \frac{1}{\sqrt{x^2+1}}\, dx = \sinh^{-1} x + C$$

$$\int \frac{1}{\sqrt{x^2-1}}\, dx = \cosh^{-1} x + C, \ |x| > 1$$

$$\int \frac{-1}{|x|\sqrt{1-x^2}}\, dx = \text{sech}^{-1} |x| + C, \ 0 < x < 1$$

$$\int \frac{-1}{|x|\sqrt{1+x^2}}\, dx = \text{csch}^{-1} |x| + C, \ x \neq 0$$

$$\int \frac{1}{1-x^2}\, dx = \begin{cases} \tanh^{-1} x + C, & |x| < 1 \\ \coth^{-1} x + C, & |x| > 1 \end{cases}$$

TAYLOR AND MACLAURIN SERIES

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$

TRIGONOMETRIC SUBSTITUTION

Expression	***Trigonometry***	***Hyperbolic***
$\sqrt{x^2+k^2}$	$x = k\tan\theta$	$x = k\sinh\theta$
$\sqrt{x^2-k^2}$	$x = k\sec\theta$	$x = k\cosh\theta$
$\sqrt{k^2-x^2}$	$x = k\sin\theta$	$x = k\tanh\theta$

FINAL EXAMINATION

SEMESTER / SESSION : SEM II / 2011/2012 | COURSE : 4 BEE

COURSE : ENGINEERING MATHEMATICS I | COURSE CODE : BSM 1913

Formulae

TRIGONOMETRIC SUBSTITUTION

$t = \tan\frac{1}{2}x$		$t = \tan x$	
$\sin x = \dfrac{2t}{1+t^2}$	$\cos x = \dfrac{1-t^2}{1+t^2}$	$\sin 2x = \dfrac{2t}{1+t^2}$	$\cos 2x = \dfrac{1-t^2}{1+t^2}$
$\tan x = \dfrac{2t}{1-t^2}$	$dx = \dfrac{2dt}{1+t^2}$	$\tan 2x = \dfrac{2t}{1-t^2}$	$dx = \dfrac{dt}{1+t^2}$

IDENTITIES OF TRIGONOMETRY AND HYPERBOLIC

Trigonometric Functions

$$\cos^2 x + \sin^2 x = 1$$
$$\sin 2x = 2\sin x\cos x$$
$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 2\cos^2 x - 1$$
$$= 1 - 2\sin^2 x$$
$$1 + \tan^2 x = \sec^2 x$$
$$1 + \cot^2 x = \csc^2 x$$
$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$
$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$
$$\sin(x \pm y) = \sin x\cos y \pm \sin y\cos x$$
$$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$$
$$2\sin ax\cos bx = \sin(a+b)x + \sin(a-b)x$$
$$2\sin ax\sin bx = \cos(a-b)x - \cos(a+b)x$$
$$2\cos ax\cos bx = \cos(a-b)x + \cos(a+b)x$$

Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2}$$
$$\cosh x = \frac{e^x + e^{-x}}{2}$$
$$\cosh^2 x - \sinh^2 x = 1$$
$$\sinh 2x = 2\sinh x\cosh x$$
$$\cosh 2x = \cosh^2 x + \sinh^2 x$$
$$= 2\cosh^2 x - 1$$
$$= 1 + 2\sinh^2 x$$
$$1 - \tanh^2 x = \text{sech}^2 x$$
$$\coth^2 x - 1 = \text{csch}^2 x$$
$$\tanh 2x = \frac{2\tanh x}{1+\tanh^2 x}$$
$$\tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x\tanh y}$$
$$\sinh(x \pm y) = \sinh x\cosh y \pm \sinh y\cosh x$$
$$\cosh(x \pm y) = \cosh x\cosh y \pm \sinh x\sinh y$$

CURVATURE, ARC LENGTH AND SURFACE AREA OF REVOLUTION

$$\kappa = \frac{\left|\dfrac{d^2y}{dx^2}\right|}{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}$$

$$\kappa = \frac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{\left[\dot{x}^2 + \dot{y}^2\right]^{3/2}}$$

$$L = \int_{x_1}^{x_2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$L = \int_{y_1}^{y_2} \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

$$S = 2\pi\int_{x_1}^{x_2} f(x)\sqrt{1+\left(\frac{d}{dx}[f(x)]\right)^2}\, dx$$

$$S = 2\pi\int_{y_1}^{y_2} g(y)\sqrt{1+\left(\frac{d}{dy}[g(y)]\right)^2}\, dy$$