UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2011/2012

COURSE NAME	:	ENGINEERING MATHEMATICS II
COURSE CODE	:	BWM 10203/ BSM 1923
PROGRAMME	:	1 BFF/BDD 2 BFF/BDD
EXAMINATION DATE	:	JUNE 2012
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER ALL QUESTIONS.

THIS EXAMINATION PAPER CONSISTS OF **SIX (6)** PAGES

Q1 A periodic function $f(x)$ is defined by

$$f(x)=\begin{cases}-4, & -\pi \le x < 0,\\ 4, & 0 \le x < \pi.\end{cases}$$

$$f(x)=f(x+2\pi).$$

which it can be written as a Fourier series

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

(a) Sketch the graph of $f(x)$ over $-3\pi \le x \le 3\pi$.

(2 marks)

(b) For $n = 1, 2, 3, \ldots$, show that

(i) $a_0 = 0.$

(ii) $a_n = 0.$

(7 marks)

(c) Calculate the Fourier coefficient, b_n.

(4 marks)

(d) Show that the Fourier series expansion for $f(x)$ is

$$f(x)=\frac{16}{\pi}\sum_{n=1}^{\infty}\frac{\sin(2n-1)x}{2n-1}.$$

(3 marks)

(e) By referring to the periodic function $f(x)$ above and use $x=\frac{\pi}{2}$, deduce the sum of the infinite series

$$1-\frac{1}{3}+\frac{1}{5}-\cdots.$$

(4 marks)

Q2 (a) The temperature distribution, $u(x, t)$ in a 5 m long metal rod governed by

$$u_t = 25u_{xx}, \ (0 < x < 5, t > 0),$$
$$u(0, t) = u(5, t) = 0$$
$$u(x, 0) = 20\sin \pi x$$

Determine the solution for temperature distribution, $u(x, t)$ at any point in the rod at time t if

$$u(x,t) = \sum_{n=1}^{\infty} e^{-\left(\frac{nc\pi}{L}\right)^2 t} A_n \sin\frac{n\pi}{L}x.$$

(7 marks)

(b) A stretched string of length 3 cm is set oscillating. The initial state is given by a function, $f(x) = x(3-x)$. The string has been released with zero initial velocity. By applying the wave equation, $u_{xx} = u_{tt}$, the subsequent displacement of the string is

$$u(x, t) = \sum_{n=1}^{\infty}\left[A_n \cos\frac{nc\pi}{L}t + B_n \sin\frac{nc\pi}{L}t\right]\sin\frac{n\pi}{L}x.$$

(i) List the initial and boundary conditions.

(ii) Calculate the value of A_n and B_n.

$$\left[\text{Hint}: A_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi}{L}x\,dx,\ B_n = \frac{2}{nc\pi}\int_0^L g(x)\sin\frac{n\pi}{L}x\,dx\right]$$

(13 marks)

Q3 (a) Solve the following first order differential equations by using the appropriate method.

(i) $2xdx - y^2dy = 0$ (write your answer in form of $y = f(x)$).

(ii) $y' + y = \sin x; \quad y(\pi) = 1.$

(10 marks)

(b) A certain radioactive material is known to decay at the rate proportional to the amount present. If initially there is 100 milligrams of the material present and after two hours it is observed that the material has lost 10 percent of its original mass. By using growth population formula, $\frac{dx}{dt} = kx$, find

(i) an expression for the mass of the material remaining at any time t,

(ii) the mass of the material after five hours,

(iii) the time at which the material has decayed to one half of its initial mass.

Write your answers in b(ii) and b(iii) in two decimal places.

(10 marks)

Q4 (a) By using the method of variation of parameters, solve

$$y'' - 2y' + 2y = e^x \tan x.$$

[Hint: $\int \sec x \, dx = \ln\left|\sec x + \tan x\right| + c$]

(10 marks)

(b) The vertical motion of a weight attached to a spring is described by the initial value problem

$$\frac{1}{4}\frac{d^2x}{dt^2} + \frac{dx}{dt} + x = 0, \qquad x(0) = 4, \quad \frac{dx}{dt} = 2.$$

(i) solve the given differential equation.

(ii) find the value of t when $\dfrac{dx}{dt} = 0$.

(iii) by using the result in Q4(b)(i), determine the maximum vertical displacement.

(10 marks)

Q5 (a) Find

(i) $\mathcal{L}\left\{\left(3t^2 - \dfrac{2}{e^{2t}}\right)^2\right\}$

(ii) $\mathcal{L}^{-1}\left\{\dfrac{\sqrt{3}}{s} + \dfrac{e^2}{s-3} - \dfrac{5}{2(s-3)^3}\right\}$

(8 marks)

(b) Consider the function

$$f(t) = \begin{cases} e^{2t} & 0 \le t < 2, \\ t-2 & t \ge 2. \end{cases}$$

(i) Write the function $f(t)$ in the form of unit step functions.

(ii) Then, find the Laplace transform of $f(t)$.

(5 marks)

(c) Solve the equation $\dfrac{d^2y}{dx^2} + y = t^4\delta(t - \sqrt{3})$ by using Laplace Transformation, given that $y(0) = 2$ and $\dot{y}(0) = 0$.

(7 marks)

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Formulae

Characteristic Equation and General Solution

Case	Roots of the Characteristic Equation	General Solution
1	m_1 and m_2 ; real and distinct	$y = Ae^{m_1 x} + Be^{m_2 x}$
2	$m_1 = m_2 = m$; real and equal	$y = (A + Bx)e^{mx}$
3	$m = \alpha \pm i\beta$; imaginary	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Undetermined Coefficients

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + \cdots + A_1 x + A_0$	$x^r(B_n x^n + \cdots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(p\cos\beta x + q\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Variation of Parameters

Wronskian	Parameter	Solution
$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$	$u_1 = -\int \frac{y_2 f(x)}{W} dx, \quad u_2 = \int \frac{y_1 f(x)}{W} dx$	$y_p = u_1 y_1 + u_2 y_2$

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Laplace Transforms

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
$t^n,\ n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
e^{at}	$\frac{1}{s-a}$	$\delta(t-a)$	e^{-as}
$\sin at$	$\frac{a}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\cos at$	$\frac{s}{s^2+a^2}$	$\int_0^t f(u)g(t-u)\,du$	$F(s).G(s)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$y(t)$	$Y(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$\dot{y}(t)$	$sY(s)-y(0)$
$e^{at}f(t)$	$F(s-a)$	$\ddot{y}(t)$	$s^2Y(s)-sy(0)-\dot{y}(0)$
$t^n f(t),\ n=1,2,3,\ldots$	$(-1)^n \frac{d^nF(s)}{ds^n}$		

Periodic Function for Laplace transform : $\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)\,dt, \quad s>0.$

Fourier Series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right]$$

where

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$$