UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2011/2012

COURSE NAME	:	ENGINEERING TECHNOLOGY MATHEMATICS II
COURSE CODE	:	BWM 11903
PROGRAMME	:	1 BDC / 1 BDM
EXAMINATION DATE	:	JUNE 2012
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS EXAMINATION PAPER CONSISTS OF **FIVE (5)** PAGES

Q1 Find the Laplace transforms of the following functions.

(a) $f(t) = (2t+5)(4-t) + e^{2t}e^{-3t}$.

(3 marks)

(b) $f(t) = t^5 e^{5t} - e^{-5t}\cos 5t$.

(4 marks)

(c) $f(t) = t^2 \sinh 4t$.

(4 marks)

(d) $f(t) = \delta(t-\pi)t\sin^2 t$.

(2 marks)

(e) $f(t) = \begin{cases} e^{3t}, & 0 < t < 4, \\ t^2, & t \geq 4. \end{cases}$

(7 marks)

Q2 (a) Given

$$\frac{s+8}{s^3 - 2s^2 - 8s} \equiv \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s-4}.$$

Calculate the value of A, B and C.

(5 marks)

(b) By using the result in Q2 (a), find the inverse Laplace transform for

(i) $\dfrac{s+8}{s^3 - 2s^2 - 8s}$.

(ii) $\dfrac{(s+8)e^{-3s}}{s^3 - 2s^2 - 8s}$.

(8 marks)

(c) Consider the initial value problem

$$y'' - 2y' - 8y = 8$$

subject to the conditions $y(0) = 0$ and $y'(0) = 1$. Determine the solution of the initial value problem by using the result in Q2 (b).

(7 marks)

Q3 A periodic function $f(x)$ is defined by

$$f(x)=\begin{cases}-4, & -\pi \le x < 0,\\ 4, & 0 \le x < \pi.\end{cases}$$
$$f(x)=f(x+2\pi).$$

which it can be written as a Fourier series

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx).$$

(a) Sketch the graph of $f(x)$ over $-3\pi \le x \le 3\pi$.

(2 marks)

(b) For $n = 1, 2, 3, \ldots$, show that

(i) $a_0 = 0$.

(ii) $a_n = 0$.

(7 marks)

(c) Calculate the Fourier coefficient, b_n.

(4 marks)

(d) Show that the Fourier series expansion for $f(x)$ is

$$f(x)=\frac{16}{\pi}\sum_{n=1}^{\infty}\frac{\sin(2n-1)x}{2n-1}.$$

(3 marks)

(e) By choosing an appropriate values for x, deduce the sum of the infinite series

$$1-\frac{1}{3}+\frac{1}{5}-\cdots.$$

(4 marks)

Q4 (a) The temperature distribution, $u(x, t)$ in a 5 m long metal rod governed by

$$u_t = 25u_{xx}, \quad (0 < x < 5, t > 0),$$

$$u(0, t) = u(5, t) = 0,$$

$$u(x, 0) = 20\sin(\pi x).$$

Determine the solution for temperature distribution, $u(x, t)$ at any point in the rod at time t if

$$u(x,t) = \sum_{n=1}^{\infty} e^{-\left(\frac{nc\pi}{L}\right)^2 t} A_n \sin\frac{n\pi}{L}x.$$

(7 marks)

(b) A stretched string of length 3 cm is set oscillating. The initial state is given by a function, $f(x) = x(x-3)$. The string has been released with zero initial velocity. By applying the wave equation, $u_{xx} = u_{tt}$, the subsequent displacement of the string is

$$u(x, t) = \sum_{n=1}^{\infty}\left[A_n \cos\frac{nc\pi}{L}t + B_n \sin\frac{nc\pi}{L}t\right]\sin\frac{n\pi}{L}x.$$

(i) List the initial and boundary conditions.

(ii) Use the Fourier series method to calculate the value of A_n and B_n.

$$\left[\text{Hint}: A_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx,\ B_n = \frac{2}{nc\pi}\int_0^L g(x)\sin\frac{n\pi x}{L}dx\right]$$

(13 marks)

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FORMULAE

Laplace Transform

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$t^n f(t)$, $n=1,2,3,\ldots$	$(-1)^n \frac{d^n F(s)}{ds^n}$
t^n, $n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	$H(t-a)$	$\frac{e^{-as}}{s}$
e^{at}	$\frac{1}{s-a}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
$\sin at$	$\frac{a}{s^2+a^2}$	$\delta(t-a)$	e^{-as}
$\cos at$	$\frac{s}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$y(t)$	$Y(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$y'(t)$	$sY(s)-y(0)$
$e^{at}f(t)$	$F(s-a)$	$y''(t)$	$s^2Y(s)-sy(0)-y'(0)$

Convolution Theorem

If $\mathcal{L}^{-1}\{F(s)\} = f(t)$ and $\mathcal{L}^{-1}\{G(s)\} = g(t)$

then $\mathcal{L}^{-1}\{F(s)G(s)\} = \int_0^t f(u)g(t-u)\,du$

Fourier Series of period 2L

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\frac{n\pi}{L}x + b_n \sin\frac{n\pi}{L}x\right]$$

where

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}dx$$