UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2011/2012

COURSE NAME	:	ENGINEERING MATHEMATICS II
COURSE CODE	:	BSM 1933
PROGRAMME	:	4 BEE
EXAMINATION DATE	**:**	**JANUARY 2012**
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER ALL QUESTIONS IN **PART A** AND **THREE (3)** QUESTIONS IN **PART B**

THIS EXAMINATION PAPER CONSISTS OF 7 PAGES

PART A

Q1 (a) Obtain the half-range cosine series for

$$f(x) = \pi - x, \quad 0 < x < \pi.$$

Expand the first three nonzero terms.

(10 marks)

(b) Using the definition of Fourier transform to find the Fourier transform of the following function.

$$f(t) = \begin{cases} t, & 0 < t < 1 \\ 0, & \text{otherwise.} \end{cases}$$

(5 marks)

(c) By using linearity and time shift, calculate the Fourier transform of

$$f(t) = 5e^{-3t}H(t-1) + 7e^{-3(t-2)}H(t-2).$$

[Hint:
Time shift for Fourier transform:
If $\mathcal{F}\{f(t)\} = F(\omega)$, then for any constant number a, $\mathcal{F}\{f(t-a)\} = e^{-i\omega a}F(\omega)$.]

(5 marks)

Q2 Given a second order ordinary differential equation

$$y'' - y = e^{2x}.$$

Assume that the solution for the differential equation is $y(x) = \sum_{m=0}^{\infty} c_m x^m$.

(a) Find $y'(x)$ and $y''(x)$.

(2 marks)

(b) Expand the series up to x^3 in each summation, and by comparing coefficients of x^0, x^1, x^2 and x^3, show that the solution is

$$y(x) = c_0\left[1 + \frac{x^2}{2} + \frac{x^4}{4!} + \cdots\right] + c_1\left[x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots\right] + \left[\frac{x^2}{2} + \frac{x^3}{3} + \frac{5x^4}{24} + \frac{x^5}{12} + \cdots\right].$$

$$\left[\text{Hint: } e^{2x} = \sum_{m=0}^{\infty} \frac{(2x)^m}{m!}\right]$$

(13 marks)

(c) Given the initial conditions for the differential equation are $y(0) = 2$ and $y'(0) = 6$. Find the particular series solution.

(5 marks)

PART B

Q3 (a) Using the substitutions $x = X + 3$ and $y = Y - 1$, show that the equation

$$\frac{dy}{dx} = \frac{2x - 3y - 9}{3x + 5y - 4}$$

can be reduced to a homogeneous equation. Hence, find the solution of the original equation.

(13 marks)

(b) Given the *R-L* circuit with source of emf, $E(t)$ as in Figure Q3 (b) below.

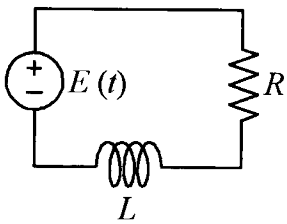

Figure Q3 (b)

The circuit has inductance $L = 5$ H, resistance $R = 15\,\Omega$, electromotive force $E(t) = 10$ V and $i(t)$ A is the current flowing in the circuit. The initial current is i_0.

(i) Show that the mathematical model for the *R-L* circuit is given by

$$\frac{di(t)}{dt} + 3i(t) = 2.$$

(ii) Find the current, $i(t)$, flowing in the circuit at time t.

(7 marks)

Q4 Given a *LC*-circuit in Figure Q4 with $L = 4$ H, $C = 0.01$ F, which is connected to a source of voltage $E(t) = 400 \sin 5t$ V.

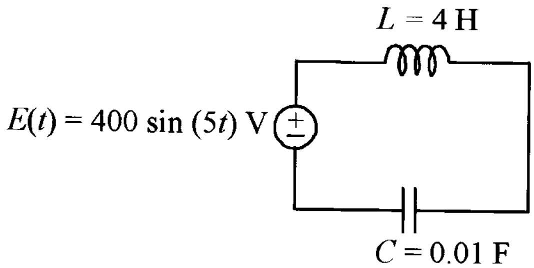

Figure Q4

(a) Show that the *LC*-circuit can be modelled by $i'' + 25\,i = 500 \cos 5t$.

(3 marks)

(b) Find the general solution to the second-order differential equation in (a).

(10 marks)

(c) Given when $t=0$, the charge, $q(0)=-1$, show that $i'(0)=25$.

(2 marks)

(d) Find the particular solution to the second-order differential equation in (a) if the current is zero when $t=0$.

(5 marks)

Q5 Given the network circuit in Figure Q5 below.

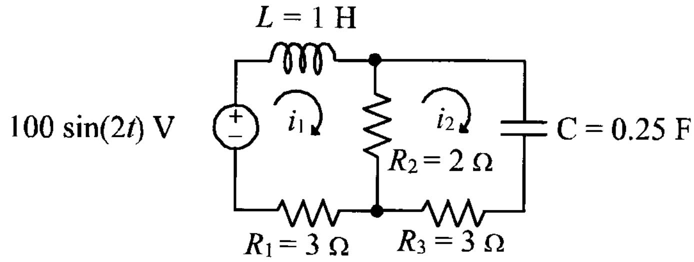

Figure Q5

(a) Show that the network circuit can be modeled by the following system of first-order differential equation

$$\begin{pmatrix} i_1' \\ i_2' \end{pmatrix} = \begin{pmatrix} -5 & 2 \\ -2 & 0 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} + \begin{pmatrix} 100\sin(2t) \\ 40\sin(2t) \end{pmatrix}.$$

(4 marks)

(b) Find the general solution to the above system of first-order differential equation by using method of undetermined coefficients.

(16 marks)

Q6 (a) Prove that

$$\mathcal{L}^{-1}\left[\frac{1-e^{-10s}}{(3s+1)(s+1)}\right]=\left[\frac{1}{2}e^{-\frac{1}{3}t}-\frac{1}{2}e^{-t}\right]-\left[\frac{1}{2}e^{-\frac{1}{3}(t-10)}-\frac{1}{2}e^{-(t-10)}\right]H(t-10).$$

(10 marks)

(b) Figure Q6 (b) below shows an RLC circuit with $L=3$ H, $R=4\,\Omega$, and $C=1$F which is initially at rest. A power source of 5V is applied to the circuit for the first 10 seconds. After 10 seconds, the power source is removed.

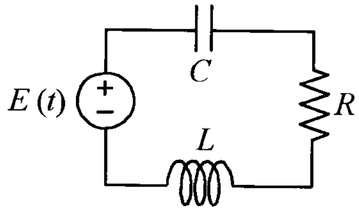

Figure Q6 (b)

(i) Show the *RLC* circuit can be governed by

$$3\frac{di}{dt}+4i+\int_0^t i(\tau)\,d\tau=5\left[1-H(t-10)\right].$$

(ii) Using the answer in **Q6** (a), find the current $i(t)$ in the circuit at any time t.

(10 marks)

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Formulae

Second-order Differential Equation

The roots of characteristic equation and the general solution for differential equation $ay'' + by' + cy = 0$.

Characteristic equation: $am^2 + bm + c = 0$.		
Case	The roots of characteristic equation	General solution
1.	Real and different roots: m_1 and m_2	$y = Ae^{m_1 x} + Be^{m_2 x}$
2.	Real and equal roots: $m = m_1 = m_2$	$y = (A + Bx)e^{mx}$
3.	Complex roots: $m_1 = \alpha + \beta i$, $m_2 = \alpha - \beta i$	$y = e^{\alpha x}(A\cos \beta x + B\sin \beta x)$

The method of undetermined coefficients for system of first order linear differential equations

For non-homogeneous for system of first order linear differential equations $\mathbf{Y}'(x) = A\,\mathbf{Y}(x) + \mathbf{G}(x)$, the particular solution $\mathbf{Y}_p(x)$ is given by:

$\mathbf{G}(x)$	$\mathbf{Y}_p(x)$	$\mathbf{G}(x)$	$\mathbf{Y}_p(x)$
$\mathbf{u}$	$\mathbf{a}$	$\mathbf{u}e^{\lambda x}$	$\mathbf{a}e^{\lambda x}$
$\mathbf{u}x + \mathbf{v}$	$\mathbf{a}x + \mathbf{b}$	$\mathbf{u}\cos \alpha x$ or $\mathbf{u}\sin \alpha x$	$\mathbf{a}\sin \alpha x + \mathbf{b}\cos \alpha x$

Laplace Transform

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\dfrac{a}{s}$	$\int_0^t f(\tau)\,d\tau$	$\dfrac{F(s)}{s}$
e^{at}	$\dfrac{1}{s-a}$	$H(t-a)$	$\dfrac{e^{-as}}{s}$
$\sin at$	$\dfrac{a}{s^2+a^2}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
$\cos at$	$\dfrac{s}{s^2+a^2}$	$\delta(t-a)$	e^{-as}
$\sinh at$	$\dfrac{a}{s^2-a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\cosh at$	$\dfrac{s}{s^2-a^2}$	$\int_0^t f(u)g(t-u)\,du$	$F(s)\cdot G(s)$
t^n, $n = 1, 2, 3, \ldots$	$\dfrac{n!}{s^{n+1}}$	$y(t)$	$Y(s)$
$e^{at}f(t)$	$F(s-a)$	$y'(t)$	$sY(s) - y(0)$
$t^n f(t)$, $n = 1, 2, 3, \ldots$	$(-1)^n \dfrac{d^n}{ds^n}F(s)$	$y''(t)$	$s^2Y(s) - sy(0) - y'(0)$

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Electrical Formula

1. Voltage drop across resistor, R (Ohm's Law): $\boxed{v_R = iR}$
2. Voltage drop across inductor , L (Faraday's Law): $\boxed{v_L = L\dfrac{di}{dt}}$
3. Voltage drop across capacitor , C (Coulomb's Law): $\boxed{v_C = \dfrac{q}{C}}$ or $\boxed{i = C\dfrac{dv_C}{dt}}$
4. The relation between current, i and charge, q: $\boxed{i = \dfrac{dq}{dt}}$.

Fourier Series

Fourier series expansion of periodic function with period $2L/2\pi$	Half Range series
$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$ $a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$ $b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$ $f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n\cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n\sin\frac{n\pi x}{L}$	$a_0 = \frac{2}{L}\int_{0}^{L} f(x)\,dx$ $a_n = \frac{2}{L}\int_{0}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$ $b_n = \frac{2}{L}\int_{0}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$ $f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n\cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n\sin\frac{n\pi x}{L}$

Table of Fourier Transform $\mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t)\,e^{-i\omega t}\,dt$

$f(t)$	$F(\omega)$	$f(t)$	$F(\omega)$
$\delta(t)$	1	$\operatorname{sgn}(t)$	$\dfrac{2}{i\omega}$
$\delta(t-\omega_0)$	$e^{-i\omega_0\omega}$	$H(t)$	$\pi\delta(\omega) + \dfrac{1}{i\omega}$
1	$2\pi\delta(\omega)$	$e^{-\omega_0 t}H(t)$ for $\omega_0 > 0$	$\dfrac{1}{\omega_0 + i\omega}$
$e^{i\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	$t^n e^{-\omega_0 t}H(t)$ for $\omega_0 > 0$	$\dfrac{n!}{(\omega_0 + i\omega)^{n+1}}$
$\sin(\omega_0 t)$	$i\pi[\delta(\omega+\omega_0) - \delta(\omega-\omega_0)]$	$e^{-at}\sin(\omega_0 t)H(t)$ for $a > 0$	$\dfrac{\omega_0}{(a+i\omega)^2 + \omega_0^2}$
$\cos(\omega_0 t)$	$\pi[\delta(\omega+\omega_0) + \delta(\omega-\omega_0)]$	$e^{-at}\cos(\omega_0 t)H(t)$ for $a > 0$	$\dfrac{a+i\omega}{(a+i\omega)^2 + \omega_0^2}$
$\sin(\omega_0 t)H(t)$	$\dfrac{\pi}{2}i[\delta(\omega+\omega_0) - \delta(\omega-\omega_0)] + \dfrac{\omega_0}{\omega_0^2 - \omega^2}$		
$\cos(\omega_0 t)H(t)$	$\dfrac{\pi}{2}[\delta(\omega+\omega_0) + \delta(\omega-\omega_0)] + \dfrac{i\omega}{\omega_0^2 - \omega^2}$		