# UNIVERSITI TUN HUSSEIN ONN MALAYSIA

## FINAL EXAMINATION
## SEMESTER II
## SESSION 2011/2012

| | | |
|---|---|---|
| COURSE NAME | : | MATHEMATICS IV |
| COURSE CODE | : | BSM 2253 |
| PROGRAMME | : | 4BVV |
| EXAMINATION DATE | : | JUNE 2012 |
| DURATION | : | 3 HOURS |
| INSTRUCTION | : | ANSWER **ALL** QUESTIONS |

THIS EXAMINATION PAPER CONSISTS OF **FIVE (5)** PAGES

**Q1** (a) Given $\sin\left(\sqrt{x}\right)+x=x^2$.

(i) Find the interval of the roots of the nonlinear equation from the following intervals:
$[1.1,1.3],[1.1,1.5]$ and $[1.1,2.0]$.

(ii) Hence, find the root of the equation by using **Bisection Method.** Do your calculations up to **3 iterations** only.

(11 marks)

(b) An engineer needs $4800\text{m}^3$, $5810\text{m}^3$ and $5690\text{m}^3$ of sand, fine gravel and coarse gravel, respectively, at a construction site. There are three sources where these materials can be obtained and the composition of the material from these sources is shown on **Table Q1**.

**Table Q1**

| | %Sand | %Fine gravel | %Coarse Gravel |
|---|---|---|---|
| Source 1 | 10 | 60 | 30 |
| Source 2 | 25 | 20 | 55 |
| Source 3 | 65 | 20 | 15 |

(i) Form a system of linear equations based on the above problem.

(ii) Hence, how many cubic meters must be hauled from each source in order to meet the engineer's needs? Use **Gauss Seidel Iteration** method to solve this problem. Do your calculations up to **3 iterations** only.

(14 marks)

**Q2** (a) A polluted lake has an initial concentration of a bacteria of $10^7$ parts/m$^3$, while the acceptable level is only $5\times10^6$ parts/m$^3$. The concentration of the bacteria will reduce as fresh water enters the lake. The differential equation that governs the concentration $C$ of the pollutant as a function of time (in weeks) is given by

$$\frac{dC}{dt}+0.06C=0, C(0)=10^7$$

Using the **Runge-Kutta 4$^{\text{th}}$ order method**, find the approximate concentration of the pollutant after 3.5 weeks. Take a step size of 3.5 weeks.

(14 marks)

(b) (i) Use the differential equation in **Q2(a)** to show that

$$C(t)=10^7 e^{-0.06t}$$

(Hint: Use **Separable Equation Method**).

(ii) Find the exact concentration of the pollutant after 3.5 weeks.

(11 marks)

**Q3** (a) Use **Simpson's $\frac{3}{8}$ Rule** to approximate the area bounded by the graph of

$$f(x)=\cos^3 x$$

between $x=0$, $x=3$ and $x$-axis. Use $n=6$. Give your answer to **3** decimal places.

(Hint: Area $=\int_{x_1}^{x_2}\left[f(x)-g(x)\right]dx$).

(10 marks)

(b) Solve the system of linear equations below by **Cholesky Method**.

$$\begin{aligned} 5x_1+2x_2 \quad\quad &= 2; \\ 2x_1+5x_2+2x_3 &= 2; \\ 2x_2+5x_3 &= 8. \end{aligned}$$

(15 marks)

**Q4** (a) Solve the differential equation $\frac{dy}{dx}=\frac{x^2+y^2}{xy}$, $y(1)=1$.

(Hint: Use **Homogeneous Equation Method**).

(12 marks)

(b)

**Table Q4**

| | | | |
|---|---|---|---|
| $x$ | 2.1 | 2.4 | 2.6 |
| $f(x)$ | 0.521 | 0.510 | 0.381 |

Based on the data in **Table Q4**,

(i) find the **Newton's interpolatory divided-difference** polynomial and estimate the approximate value for $f(2.5)$.

(ii) find $f(2.5)$ if $f(2.0)=0.510$ is added in the data given in **Table Q4**.

(13 marks)

**FINAL EXAMINATION**

SEMESTER / SESSION: SEM II/ 2011/2012 PROGRAMME : 4 BVV

COURSE : MATHEMATICS IV COURSE CODE : BSM 2253

**FORMULAS**

**Nonlinear equations**

Bisection : $c_i = \dfrac{a_i + b_i}{2}$, $i = 0,1,2,...$

Newton-Raphson method : $x_{i+1} = x_i - \dfrac{f(x_i)}{f'(x_i)}$, $i = 0,1,2,\ldots$

**System of linear equations**:

Gauss-Seidel Iteration:

$$x_1^{(k+1)} = \frac{b_1 - a_{12}x_2^{(k)} - a_{13}x_3^{(k)}}{a_{11}}$$

$$x_2^{(k+1)} = \frac{b_2 - a_{21}x_1^{(k+1)} - a_{23}x_3^{(k)}}{a_{22}}$$

$$x_3^{(k+1)} = \frac{b_3 - a_{31}x_1^{(k+1)} - a_{32}x_2^{(k+1)}}{a_{33}}$$

**Ordinary differential equations**

**Initial value problems:**

Fourth-order Runge-Kutta Method: $y_{i+1} = y_i + \dfrac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

where $k_1 = hf(x_i, y_i)$ $\qquad k_2 = hf(x_i + \dfrac{h}{2}, y_i + \dfrac{k_1}{2})$

$k_3 = hf(x_i + \dfrac{h}{2}, y_i + \dfrac{k_2}{2})$ $\qquad k_4 = hf(x_i + h, y_i + k_3)$

**FINAL EXAMINATION**

SEMESTER / SESSION: SEM II/ 2011/2012 PROGRAMME : 4 BVV

COURSE : MATHEMATICS IV COURSE CODE : BSM 2253

**Interpolation**

Newton Divided-difference Method

$$P_n(x) = f_0^{[0]} + f_0^{[1]}(x - x_0) + f_0^{[2]}(x - x_0)(x - x_1) + \ldots + f_0^{[n]}(x - x_0)(x - x_1)\ldots(x - x_{n-1})$$

| $i$ | $x_i$ | $f(x_i) = f_i^{[0]}$ | $f_i^{[1]}$ | $f_i^{[2]}$ | $f_i^{[3]}$ |
|---|---|---|---|---|---|
| 0 | $x_0$ | $f[x_0]$ | $f[x_0, x_1] = \dfrac{f[x_1] - f[x_0]}{x_1 - x_0}$ | $f[x_0, x_1, x_2] = \dfrac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0}$ | $f[x_0, x_1, x_2, x_3] = \dfrac{f[x_1, x_2, x_3] - f[x_0, x_1, x_2]}{x_3 - x_0}$ |
| 1 | $x_1$ | $f[x_1]$ | $f[x_1, x_2] = \dfrac{f[x_2] - f[x_1]}{x_2 - x_1}$ | $f[x_1, x_2, x_3] = \dfrac{f[x_2, x_3] - f[x_1, x_2]}{x_3 - x_1}$ | |
| 2 | $x_2$ | $f[x_2]$ | $f[x_2, x_3] = \dfrac{f[x_3] - f[x_2]}{x_3 - x_2}$ | | |
| 3 | $x_3$ | $f[x_3]$ | | | |

**Numerical Integration:**

Simpson's $\frac{1}{3}$ rule: $\int_a^b f(x)dx \approx \frac{h}{3}\left[ f_0 + f_n + 4\sum_{\substack{i=1 \\ i \text{ odd}}}^{n-1} f_i + 2\sum_{\substack{i=2 \\ i \text{ even}}}^{n-2} f_i \right]$

Simpson's $\frac{3}{8}$ rule:

$$\int_a^b f(x)dx \approx \frac{3}{8}h[f_0 + f_n + 3(f_1 + f_2 + f_4 + f_5 + \ldots + f_{n-2} + f_{n-1}) + 2(f_3 + f_6 + \ldots + f_{n-6} + f_{n-3})]$$