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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2011/2012

COURSE NAME	:	ENGINEERING MATHEMATICS III
COURSE CODE	:	BWM20403/BSM2913
PROGRAMME	:	2 BFF/BEE/BDD
EXAMINATION DATE	:	JANUARY 2012
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS EXAMINATION PAPER CONSISTS OF SIX (6) PAGES

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Q1 Let $f(x,y) = \frac{4x^2y}{x^3+y^3}$

(a) Find domain and range of $f(x)$

(2 marks)

(b) Determine whether $f(x)$ is continue at (0,0) or not. Give any reason.

(5 marks)

Q2 (a) Let $W = x + y^2 + z^3$, where $x = \ln rst$, $y = e^{-rs}$ and $z = rst$. Find W_s.

(5 marks)

(b) If $z = \frac{x+y}{\sqrt{x^2+y^2}}$, shows that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0$.

(5 marks)

Q3 Find the local extremums of the function

$$f(x,y) = 2x^3 + y^3 + 3x^2 - 3y - 12x - 4.$$

(12 marks)

Q4 Use double integrals to find the surface area of $z = \sqrt{x^2+y^2}$ that lies under the plane $z = 3$.

(7 marks)

Q5 Line integral of $f(x,y,z)$ respects to arc length S around curve C, $\int_C f(x,y,z)dS$, can be used to compute the mass and centre of mass of a thin wire when f is assumed as a density of the wire.

Consider a wire with a shape of helix $x = 2\cos t$, $y = 2\sin t$, $z = t$ for $0 \leq t \leq 2\pi$, and the density of the wire, $\delta(x,y,z) = 2$ at any point (x,y,z).

By using line integral, find the mass and the centre of mass of this wire. Does the centre of the mass lies on the curve or the wire itself?

(11 marks)

Q6 (a) Find a vector-valued function $\boldsymbol{r}(t)$ that represents the curve of intersection of the cylinder $x^2 + y^2 = 1$ and the plane $x + z = 1$.

(b) Find the unit tangent vector $\boldsymbol{T}(t)$.

(7 marks)

Q7 S be the sphere with radius 4, H is the hemisphere of S on positive value of z-axis and D is the disk $x^2 + y^2 \leq 16$ in the xy-plane. G is the solid bounded by surfaces H and D oriented outward, and $\boldsymbol{F}(x,y,z) = 2x\boldsymbol{i} + 2y\boldsymbol{j} + 5\boldsymbol{k}$ is the vector field across the surface G.

(a) Write an equation of H as $z = f(x,y)$ and plot graph G.

(3 marks)

(b) Find f_x and f_y at point $(2, 2, \sqrt{8})$ and gives its geometrical interpretation.

(6 marks)

(c) From (b), find the directional derivative of f at point $(2, 2, \sqrt{8})$ in the direction

(i) $\boldsymbol{a} = \boldsymbol{i}$

(ii) $\boldsymbol{b} = \boldsymbol{j}$

(iii) $\boldsymbol{c} = \boldsymbol{i} + \boldsymbol{j}$

(5 marks)

(d) Find the volume of G by using:

(i) double integral in cylindrical coordinate

(ii) triple integral in spherical coordinate

(7 marks)

(e) Show that the area of D is 16π unit^2 by using Green theorem to the line integral $\oint_C \frac{1}{2}(-ydx + xdy)$, where C is a simple closed path oriented counterclockwise given by D.

(4 marks)

(f) Hence, from all above related results:

(i) Use Stoke's theorem to evaluate $\oint_C \boldsymbol{F} \cdot \boldsymbol{dr}$ where C is the boundary of G in the xy-plane. What can you conclude?

(ii) Verify the Gauss theorem

(21 marks)

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Formulae

Polar coordinate: $x = r\cos\theta$, $y = r\sin\theta$, $\theta = \tan^{-1}(y/x)$, and $\iint_R f(x,y)dA = \iint_R f(r,\theta)\, r\, dr\, d\theta$

Cylindrical coordinate: $x = r\cos\theta$, $y = r\sin\theta$, $z = z$, $\iiint_G f(x,y,z)dV = \iiint_G f(r,\theta,z)\, r\, dz\, dr\, d\theta$

Spherical coordinate: $x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$, $z = \rho\cos\phi$, $x^2+y^2+z^2=\rho^2$, $0 \le \theta \le 2\pi$, $0 \le \phi \le \pi$, and

$$\iiint_G f(x,y,z)dV = \iiint_G f(\rho,\phi,\theta)\rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$$

Directional derivative: $D_{\mathbf{u}} f(x,y) = (f_x\mathbf{i} + f_y\mathbf{j})\cdot\mathbf{u}$

Let $\mathbf{F}(x,y,z) = M\,\mathbf{i} + N\,\mathbf{j} + P\,\mathbf{k}$ is vector field, then

the **divergence** of $\mathbf{F} = \nabla\cdot\mathbf{F} = \dfrac{\partial M}{\partial x} + \dfrac{\partial N}{\partial y} + \dfrac{\partial P}{\partial z}$

the **curl** of $\mathbf{F} = \nabla\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ M & N & P \end{vmatrix} = \left(\dfrac{\partial P}{\partial y} - \dfrac{\partial N}{\partial z}\right)\mathbf{i} - \left(\dfrac{\partial P}{\partial x} - \dfrac{\partial M}{\partial z}\right)\mathbf{j} + \left(\dfrac{\partial N}{\partial x} - \dfrac{\partial M}{\partial y}\right)\mathbf{k}$

Let C is a smooth curve given by $\mathbf{r}(t) = x(t)\,\mathbf{i} + y(t)\,\mathbf{j} + z(t)\,\mathbf{k}$, t is parameter, then

the unit tangent vector: $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

the unit normal vector: $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

the binormal vector: $\mathbf{B}(t) = \mathbf{T}(t)\times\mathbf{N}(t)$

the curvature: $\kappa = \dfrac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|} = \dfrac{\|\mathbf{r}'(t)\times\mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$

the radius of curvature: $\rho = 1/\kappa$

Green Theorem: $\oint_C M\,dx + N\,dy = \iint_R \left(\dfrac{\partial N}{\partial x} - \dfrac{\partial M}{\partial y}\right)dA$

Gauss Theorem: $\iint_S \mathbf{F}\bullet\mathbf{n}\,dS = \iiint_G \nabla\bullet\mathbf{F}\,dV$

Stokes' Theorem: $\oint_C \mathbf{F}\bullet d\mathbf{r} = \iint_S (\nabla\times\mathbf{F})\bullet\mathbf{n}\,dS$

Arc length

If $\mathbf{r}(t) = x(t)\,\mathbf{i} + y(t)\,\mathbf{j}$, $t \in [a,b]$, then the **arc length** $s = \int_a^b \|\mathbf{r}'(t)\|\,dt = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2}\,dt$

If $\mathbf{r}(t) = x(t)\,\mathbf{i} + y(t)\,\mathbf{j} + z(t)\,\mathbf{k}$, $t \in [a,b]$, then the **arc length** $s = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}\,dt$

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Tangent Plane

$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$

Extreme of two variable functions

$G(x, y) = f_{xx}(x, y) f_{yy}(x, y) - (f_{xy}(x, y))^2$

Case1: If $G(a,b) > 0$ and $f_{xx}(x, y) < 0$ then f has local maximum at (a,b)

Case2: If $G(a,b) > 0$ and $f_{xx}(x, y) > 0$ then f has local minimum at (a,b)

Case3: If $G(a,b) < 0$ then f has a saddle point at (a,b)

Case4: If $G(a,b) = 0$ then no conclusion can be made.

In 2-D: Lamina

Mass: $m = \iint_R \delta(x, y)\,dA$, where $\delta(x, y)$ is a density of lamina.

Moment of mass: (i) about y-axis, $M_y = \iint_R x\delta(x, y)\,dA$, (ii) about x-axis, $M_x = \iint_R y\delta(x, y)\,dA$

Centre of mass, $(\bar{x}, \bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m}\right)$

Moment inertia: (i) $I_y = \iint_R x^2\delta(x, y)\,dA$, (ii) $I_x = \iint_R y^2\delta(x, y)\,dA$, (iii) $I_o = \iint_R (x^2 + y^2)\delta(x, y)\,dA$

In 3-D: Solid

Mass, $m = \iiint_G \delta(x, y, z)\,dV$. If $\delta(x, y, z) = c$, c is a constant, then $m = \iiint_G dA$ is volume.

Moment of mass

(i) about yz-plane, $M_{yz} = \iiint_G x\delta(x, y, z)\,dV$

(ii) about xz-plane, $M_{xz} = \iiint_G y\delta(x, y, z)\,dV$

(iii) about xy-pane, $M_{xy} = \iiint z\delta(x, y, z)\,dV$

Centre of gravity, $(\bar{x}, \bar{y}, \bar{z}) = \left(\frac{M_{yz}}{m}, \frac{M_{xz}}{m}, \frac{M_{xy}}{m}\right)$

Moment inertia

(i) about x-axis: $I_x = \iiint_G (y^2 + z^2)\delta(x, y, z)\,dV$

(ii) about y-axis: $I_y = \iiint_G (x^2 + z^2)\delta(x, y, z)\,dV$

(iii) about z-axis: $I_z = \iiint_G (x^2 + y^2)\delta(x, y, z)\,dV$