



UNIVERSITI TUN HUSSEIN ONN MALAYSIA

**FINAL EXAMINATION
SEMESTER I
SESSION 2011/2012**

COURSE NAME : ENGINEERING MATHEMATICS IV

COURSE CODE : BWM 30603/BSM 3913

**PROGRAMME : 1 BEE
2 BDD/BEE/BFF
3 BDD/BEE/BFF
4 BDD/ BFF**

EXAMINATION DATE : JANUARY 2012

DURATION : 3 HOURS

**INSTRUCTION : ANSWER ALL QUESTIONS IN PART A
AND TWO (2) QUESTIONS IN PART B.**

**ALL CALCULATIONS AND ANSWERS
MUST BE IN THREE (3) DECIMAL
PLACES.**

THIS EXAMINATION PAPER CONSISTS OF SEVEN (7) PAGES

PART A

Q1 (a) Consider the heat conduction equation

$$\frac{\partial}{\partial t}T(x,t) = \alpha \frac{\partial^2}{\partial x^2}T(x,t), \quad 0 < x < 10, \quad t > 0,$$

where α is thermal diffusivity = 10, since $\alpha = c^2$.

Given the boundary conditions,

$$T(0,t) = 0, \quad T(10,t) = 100$$

and initial condition,

$$T(x,0) = x^2.$$

By using explicit finite-difference method, find $T(x,0.055)$ and $T(x,0.11)$ with 5 grid intervals on the x coordinate.

(10 marks)

(b) Let $y(x,t)$ denotes displacement of a vibrating string. If T is the tension in the string, ω is the weight per unit length, and g is acceleration due to gravity, then y satisfies the equation

$$\frac{\partial^2 y}{\partial t^2} = \frac{Tg}{\omega} \frac{\partial^2 y}{\partial x^2}, \quad 0 < x < 2, \quad t > 0.$$

Suppose a particular string is 2 m long and is fixed at both ends. By taking $T = 1.5\text{N}$, $\omega = 0.01 \text{ kg/m}$ and $g = 10 \text{ m/s}^2$, use the finite-difference method to solve for y up to level 2 only. The initial conditions are

$$y(x,0) = \begin{cases} \frac{x}{2}, & 0 \leq x \leq 1 \\ \frac{2-x}{2}, & 1 \leq x \leq 2 \end{cases} \quad \text{and} \quad \frac{\partial y}{\partial t}(x,0) = x(x-2).$$

Performed all calculations with $\Delta x = 0.5 \text{ m}$ and $\Delta t = 0.01 \text{ s}$.

(15 marks)

Q2 The steady state temperature distribution of heated rod follows the one-dimensional form of Poisson's equation

$$\frac{d^2 T}{dx^2} + Q(x) = 0.$$

Solve the above equation for a 6 cm rod with boundary conditions of $T(0,t) = 10$ and $T(6,t) = 50$ and a uniform heat source $Q(x) = 40$ with 3 equal-size elements of length by using finite-element method with linear approximation.

(25 marks)

PART B

Q3 (a) Given $f(x) = 7e^{-x} \sin x - 1$ for $-1 \leq x \leq 1$.

- (i) Find the root of $f(x)$ by using Secant method (iterate until $|f(x_i)| < \varepsilon = 0.005$).
- (ii) If the exact solution of $f(x)$ is 0.118, find the absolute error.

(8 marks)

(b) A mixture company has three sizes of packs of nuts. The *Large* size contains 2 kg of walnuts, 2 kg of peanuts and 1 kg of cashews. The *Mammoth* size contains 3 kg of walnuts, 6 kg of peanuts and 2 kg of cashews. The *Giant* size contains 1 kg of walnuts, 4 kg of peanuts and 2 kg of cashews. Suppose that the company receives an order for 14 kg of walnuts, 26 kg of peanuts and 12 kg of cashews.

- (i) By taking a , b and c represent *Large*, *Mammoth* and *Giant* size, obtain the system of linear equations for this company.
- (ii) By using Gauss elimination method, determine how can this company fill this order with the given sizes of packs.
- (iii) Suppose that the mixture company above is planning to expand their mixing productions. Their planning can be summarized as **Table Q3(b)** below:

	Walnuts	Peanuts	Cashews	Almond
Large	10	4	2	2
Mammoth	1	5	11	1
Giant	4	14	2	1
Small	3	3	2	9

Table Q3(b)

If the company receives a new order for 13 kg of walnuts, 34 kg of peanuts, 15 kg of cashews and 28 kg of Almond, determine the possible solutions of the system by using Gauss-Seidel iteration method.

(17 marks)

Q4 (a) A certain lab experiment produced the following data (**Table Q4(a)**):

x	$y(x)$
0	-100
20	280
40	1460
60	3440
80	6220

Table Q4(a)

Predict $y(x)$, when $x = 70$ by using

- (i) Lagrange polynomial interpolation and
- (ii) Newton divided-difference interpolation.

(16 marks)

(b) Evaluate $\int_0^1 e^{x^2} \sin x \, dx$ by using 3-point Gauss Quadrature.

(9 marks)

Q5 (a) Given the matrix

$$A = \begin{pmatrix} 3 & 4 & 1 \\ 4 & 3 & 0 \\ 1 & 4 & 3 \end{pmatrix}$$

- (i) By using the power method, compute the dominant eigenvalue, λ_{Largest} of A and its corresponding eigenvector v_1 .
- (ii) Then, find smallest eigenvalue, $\lambda_{\text{Smallest}}$ of A by using the shifted power method.

For those both calculations, use initial eigenvector, $v^{(0)} = (1 \ 1 \ 1)^T$ and stop the iteration when $|m_{k+1} - m_k| < \varepsilon = 0.005$.

(8 marks)

- (b) (i) The differential equation

$$\frac{dp}{dt} = rb(1-p), \quad p(0) = p_0,$$

is a model for studying the proportion, $p(t)$ of nonconformist in a society. The birth rate is b and the rate at which offspring would become nonconformist when at least one of their parents was a conformist is r . If $p(0) = 0.01$, $b = 0.02$ and $r = 0.1$, approximate $p(3)$ by using second-order Taylor series method and modified Euler's method. Assume that $h = \Delta t = 1$ year.

- (ii) Solve the following boundary value problem

$$y'' + 4y' + 4y = e^{-t}, \quad y(0) = 0, y(1) = 0.$$

Consider $h = \Delta t = 0.25$.

(17 marks)

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FORMULAS**Nonlinear equations**

Secant method : $x_{i+2} = \frac{x_i f(x_{i+1}) - x_{i+1} f(x_i)}{f(x_{i+1}) - f(x_i)}$

System of linear equations

Gauss-Seidel iteration method:

$$x_i^{(k+1)} = \frac{b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^n a_{ij} x_j^{(k)}}{a_{ii}}, i = 1, 2, \dots, n$$

Interpolation

Lagrange polynomial: $P_n(x) = \sum_{i=0}^n L_i(x) f(x_i), i = 0, 1, 2, \dots, n$ where $L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^n \frac{(x - x_j)}{(x_i - x_j)}$

Newton divided difference:

$$P_n(x) = f_0^{[0]} + f_0^{[1]}(x - x_0) + f_0^{[2]}(x - x_0)(x - x_1) + \dots + f_0^{[n]}(x - x_0)(x - x_1) \dots (x - x_{n-1})$$

Numerical differentiation and integrationIntegration:

Gauss quadrature:

For $\int_a^b f(x) dx$, $x = \frac{(b-a)t + (b+a)}{2}$

3-points: $\int_{-1}^1 f(x) dx \approx \frac{5}{9} g\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9} g(0) + \frac{5}{9} g\left(\sqrt{\frac{3}{5}}\right)$

Eigenvalue

Power Method: $v^{(k+1)} = \frac{1}{m_{k+1}} A v^{(k)}, k = 0, 1, 2, \dots$

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Ordinary differential equationsInitial value problems:

Modified Euler's method: $y_{i+1} = y_i + \frac{1}{2}k_1 + \frac{1}{2}k_2$

where $k_1 = hf(x_i, y_i)$, $k_2 = hf(x_i + h, y_i + k_1)$

Second order Taylor series method: $y(x_{i+1}) = y(x_i) + hy'(x_i) + \frac{h^2}{2!}y''(x_i)$

Boundary value problems:

Finite difference method:

$$y'_i \approx \frac{y_{i+1} - y_{i-1}}{2h}, \quad y''_i \approx \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$$

Partial differential equation

Heat Equation: Finite difference method

$$\left(\frac{\partial u}{\partial t}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \quad \frac{u_{i,j+1} - u_{i,j}}{k} = c^2 \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2}$$

Wave equation: Finite difference method

$$\left(\frac{\partial^2 u}{\partial t^2}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \quad \frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{k^2} = c^2 \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

Finite element method

$$KT = F_b - F_l,$$

where $K_{ij} = A_{ij} = \int_p^q a \frac{dN_i}{dx} \frac{dN_j}{dx} dx$ is stiffness matrix,

$$T = T_i \quad , \quad F_b = \left[N_i a \frac{dT}{dx} \right]_p^q \quad , \quad F_l = \int_p^q N_i f(x) dx$$