# UNIVERSITI TUN HUSSEIN ONN MALAYSIA

## FINAL EXAMINATION
## SEMESTER I
## SESSION 2012/2013

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|---|---|---|
| COURSE NAME | : | ENGINEERING MATHEMATICS IV |
| COURSE CODE | : | BWM 30602 |
| PROGRAMME | : | 2 BEE<br>3 BEE |
| EXAMINATION DATE | : | JANUARY 2013 |
| DURATION | : | 2 HOURS AND 30 MINUTES |
| INSTRUCTION | : | ANSWER **ALL** QUESTIONS<br>ALL CALCULATIONS AND ANSWERS MUST BE IN THREE (3) DECIMAL PLACES. |

THIS QUESTION PAPER CONSISTS OF **FOUR (4)** PAGES

Q1 (a) Find the root of $\sin(2x) + x^3 - 3 = 0$ by using secant method at the interval $[1, 2]$. Iterate until $|f(x_i)| < \varepsilon = 0.005$.

(12 marks)

(b) Given the system of linear equations

$$6x_1 - 2x_2 + x_3 = 11$$
$$-2x_1 + 7x_2 + 2x_3 = 5$$
$$x_1 + 2x_2 - 5x_3 = -1$$

Solve the above system by using Gauss-Seidel iteration method up to 4 iterations only.

(13 marks)

Q2 (a) Given matrix $A$ defined by

$$A = \begin{bmatrix} 4 & 2 & 0 \\ 1 & 4 & 2 \\ 0 & 1 & 4 \end{bmatrix}$$

Find the dominant eigenvalue and its corresponding eigenvector for matrix $A$ by using power method. Use initial guess for eigenvector, $v^{(0)} = (1 \quad 1 \quad 1)^T$. Calculate until $|m_{k+1} - m_k| < 0.005$.

(15 marks)

(b) Find the approximate value of $\int_0^1 \frac{4}{1+x^2}\,dx$ by using $\frac{1}{3}$ Simpson's rule with step size, $h = 0.1$.

(10 marks)

Q3 (a) Apply fourth-order Runge-Kutta method (RK4) to solve the following initial value problem

$$\frac{dy}{dx} = -2x - y,\ 0 \le x \le 0.3$$

with initial value $y(0) = -1$ and step size, $h = 0.1$.

(10 marks)

(b) Solve the boundary-value problem of

$$y'' - \left(1 - \frac{x}{5}\right) y = x,\ \ y(1) = 2 \text{ and } y(3) = -1$$

by using the finite-difference method with grid size, $h = \Delta x = 0.5$.

(15 marks)

Q4 (a) Let $u(x,t)$ be the displacement of uniform wire which is fixed at both ends along $x$-axis at time $t$. The distribution of $u(x,t)$ is given by the wave equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2},\ 0 < x < 1,\ t > 0$$

with the boundary conditions $u(0,t) = u(1,t) = 0$ and the initial conditions $u(x,0) = \sin 4\pi x$, $\frac{\partial u}{\partial t}(x,0) = 0$ for $0 \le x \le 1$. Solve the wave equation up to level $t = 0.1$ by using finite-difference method with $\Delta x = h = 0.25$ and $\Delta t = k = 0.1$.

(10 marks)

(b) Given the heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < 1$, and $t > 0$ with the boundary conditions $u(0,t) = 20t^2$, $u(1,t) = 10t$ and the initial condition $u(x,0) = x(1-x)$. Solve the heat equation up to level $t = 0.3$ by using finite-difference method with $\Delta x = h = 0.5$ and $\Delta t = k = 0.1$.

(15 marks)

**- END OF QUESTION -**

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SEMESTER / SESSION: SEM I/ 2012/2013 PROGRAMME: 2/3 BEE
COURSE: ENGINEERING MATHEMATICS IV CODE : BWM 30602

## FORMULAS

Secant method $x_{i+2} = \dfrac{x_i f(x_{i+1}) - x_{i+1} f(x_i)}{f(x_{i+1}) - f(x_i)}$, $i = 0,1,2,3,...$

Gauss-Seidel iteration method $x_i^{(k+1)} = \dfrac{b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)}}{a_{ii}}, i = 1,2,\ldots,n$

Lagrange polynomial

$P_n(x) = \sum_{i=0}^{n} L_i(x) f(x_i), i = 0,1,2,\ldots,n$ where $L_i(x) = \prod_{\substack{j=0 \\ j\neq i}}^{n} \dfrac{(x - x_j)}{(x_i - x_j)}$

3-point central difference: $f'(x) \approx \dfrac{f(x+h) - f(x-h)}{2h}$

$$f''(x) \approx \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

Simpson's $\frac{1}{3}$ rule: $\int_a^b f(x)dx \approx \dfrac{h}{3}\left[ f_0 + f_n + 4\sum_{\substack{i=1 \\ i \text{ odd}}}^{n-1} f_i + 2\sum_{\substack{i=2 \\ \text{even}}}^{n-2} f_i \right]$

Power Method $v^{(k+1)} = \dfrac{1}{m_{k+1}} A v^{(k)}, \quad k = 0,1,2,...$

Classical 4th order Runge-Kutta method. $y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

where $k_1 = hf(x_i, y_i)$ $\qquad k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2})$

$k_3 = hf(x_i + \frac{h}{2}, y_i + \frac{k_2}{2})$ $\qquad k_4 = hf(x_i + h, y_i + k_3)$

Boundary value problems

$y_i' \approx \dfrac{y_{i+1} - y_{i-1}}{2h}$, $\qquad y_i'' \approx \dfrac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$

Finite difference method:

$$\left(\frac{\partial^2 u}{\partial t^2}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \quad \Leftrightarrow \quad \frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{k^2} = c^2 \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

$$\frac{\partial u(x,0)}{\partial t} = \frac{u_{i,j+1} - u_{i,j-1}}{2k} = g(x_i)$$

$$\left(\frac{\partial u}{\partial t}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \quad \Leftrightarrow \quad \frac{u_{i,j+1} - u_{i,j}}{k} = c^2 \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$