UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2012/2013

COURSE NAME	:	ENGINEERING MATHEMATICS IV
COURSE CODE	:	BWM 30603 / BSM 3913
PROGRAMME	:	2 BDD 3 BDD 4 BDD / BEE / BFF
EXAMINATION DATE	:	DECEMBER 2012 / JANUARY 2013
DURATION	:	3 HOURS
INSTRUCTIONS	:	ANSWER ALL QUESTIONS IN **PART A** AND **TWO (2)** QUESTIONS IN **PART B.** ALL CALCULATIONS AND ANSWERS MUST BE IN THREE (3) DECIMAL PLACES.

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

PART A

Q1 A straight steel bar of length 40 mm is heated for 8 seconds at its ends from its room Temperature of 25 °C. The heating process is going up slowly as illustrated in **Figure Q1** below.

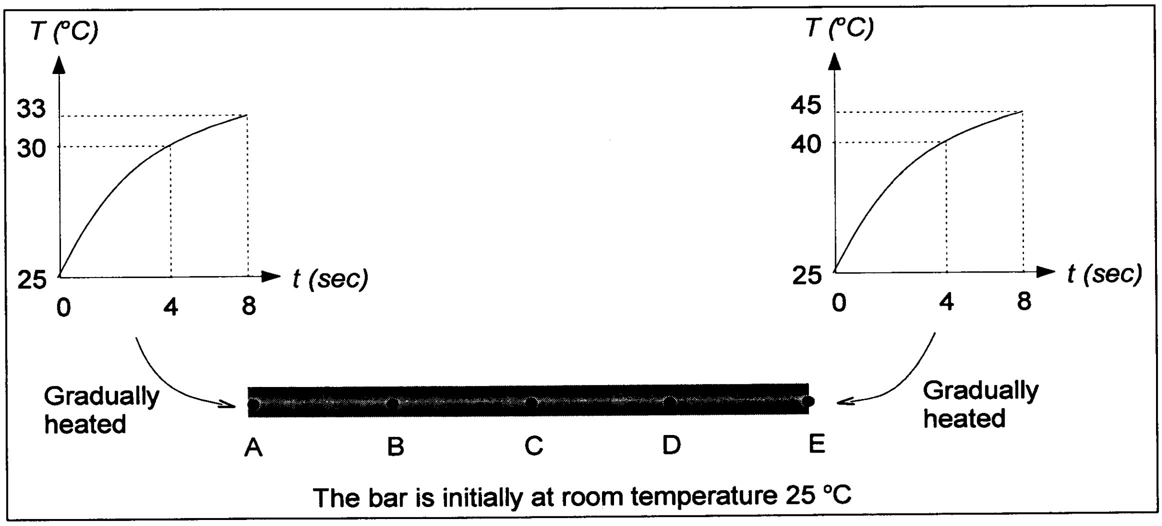

Figure Q1: Bar Under Gradually Heating

The unsteady state heating equation follows a heat equation

$$\frac{\partial T}{\partial t} - K\frac{\partial^2 T}{\partial x^2} = 0$$

where K is thermal diffusivity of steel material and x is the longitudinal coordinate of the bar. The thermal diffusivity of the steel is given as 10 mm^2/s, $\Delta t = k = 4$ and $\Delta x = h = 10$.

(a) Write down the partial differential equation of the heat equation in explicit finite-difference form.

(4 marks)

(b) Simplify the finite-difference form that you have obtained in Q1(a).

(4 marks)

(c) Draw the finite-difference grid to predict the temperature of point A, B, C, D, E up to 8 seconds (second level). The control points (A, B, C, D, E) are equally distributed along x-axis.

(5 marks)

(d) Hence, solve the heat equation by using explicit finite-difference method.

(12 marks)

Q2 A mechanical structure consists of a rod and a heavy spring. The rod (with the cross section, A = 0.002 m^2 and modulus of elasticity, E = 200 GPa) and the spring (with the spring constant, k = 40 MN/m) are attached together. The end of the rod that is not connected to the spring is rigidly fixed to the wall. At the connection point to the spring, the rod is axially pulled, F = 1000 kN as shown in **Figure Q2** below. In front of the spring there is a fixed wall, so that the end spring will deform maximum 2 mm.

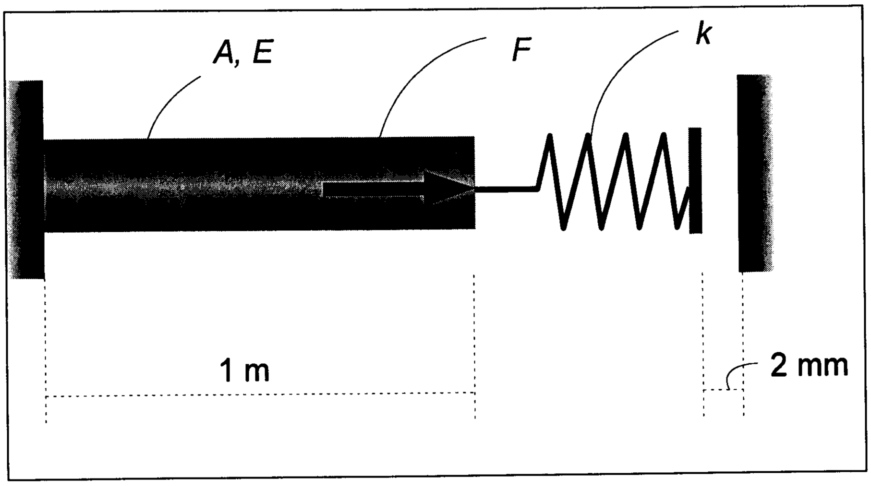

Figure Q2: Axial Structure Under Loading

(a) Draw the finite element model consists of 2 elements. Indicate clearly the node and element numbers. Specify also the constraints (boundary conditions) of your model (5 marks)

(b) Find the stiffness matrix of each element. (5 marks)

(c) Determine the global stiffness matrix of the structure before and after constraints (you can use either direct elimination method or penalty method). (10 marks)

(d) Find the deformation of each node. (5 marks)

PART B

Q3 (a) Find a positive root of nonlinear equation $x^3 - 2x - 5 = 0$ by using Secant method in interval $[2,3]$.

(5 marks)

(b) (i) Solve the following system of linear equations by using Gauss-Seidel iteration method. Use $x^{(0)} = 113.712$, $y^{(0)} = 55.123$ and $z^{(0)} = 34.008$.

$$\begin{aligned} x + 2y + 4z &= 360 \\ 2x + 6x + 3z &= 660 \\ 4x + 2y + z &= 600 \end{aligned}$$

(ii) Solve the following system of linear equations by Thomas Algorithm.

$$\begin{aligned} x_1 + 2x_2 \quad &= 9 \\ 6x_1 + 6x_2 - 8x_3 &= 1 \\ -3x_2 + x_3 &= 0 \end{aligned}$$

(20 marks)

Q4 The differential equation for steady state condition of heat conduction through a wall with considering internal heat generation is given by

$$\frac{d^2T}{dx^2} + \frac{G}{k} = 0 ,$$

where $T =$ temperature $(°C)$, $x =$ position (cm), $G =$ internal heat source (W/cm^3) and $k =$ thermal conductivity $(\text{W/cm/}°C)$. An experimental work was done to prove this relationship. Temperatures of the wall at specific positions were measured and tabulated in **Table Q4(a)**.

Table Q4(a)

x (cm)	-3.00	-2.25	-1.50	-0.75	0.75	1.50	2.25	3.00
T (°C)	40.00	42.81	44.82	46.03	46.03	44.82	42.81	40.00

The thermal conductivity of the wall is 0.21 and the cross-sectional area of the wall, A is 150,000 cm^2. At the same time the heat flux, q at the wall surface was hourly measured from 9.00 am till 5.00 pm by a solar collector. The data recorded are listed in **Table Q4(b)** as follows:

Table Q4(b)

t (hour)	0	1	2	3	4	5	6	7	8
q (Cal./cm^2/hour)	1.62	5.32	6.29	7.80	8.81	8.00	8.57	8.03	7.04

(a) Refer **Table Q4(a)**. Find the Newton's interpolatory divided-difference polynomial, $P_3(x)$ by using data from $x=-1.5$ to $x=1.5$, and predict the temperature at $x=0$.

(7 marks)

(b) By using 5-point difference formula for second derivative with the result from Q4(a) and data from **Table Q4(a)**,

(i) show that the temperature is maximum at $x = 0$.
(ii) determine the internal heat source, G at $x = 0$.

(10 marks)

(c) Refer **Table Q4(b).** Estimate the total heat absorbed by the wall from 9.00 am to 5.00 pm by using Simpson's ⅓ rule, if the total heat absorbed, H is given by

$$H = 0.45 \int_0^t qA \, dt .$$

(8 marks)

Q5 (a) Determine the largest eigenvalue and its corresponding eigenvector for matrix

$$A = \begin{pmatrix} 2 & 2 & 10 \\ 8 & 3 & 4 \\ 10 & 4 & 5 \end{pmatrix}$$

by using power method. Use the trial value $v^{(0)} = \begin{pmatrix} 1 & 0 & 1 \end{pmatrix}^T$ and calculate until $|m_{k+1} - m_k| < 0.005$.

(7 marks)

(b) Given the initial-value problem (IVP) as follows:

$$\frac{dy}{dt} = \frac{y^2}{t+2} \quad \text{at } t = 0(0.2)0.6.$$

Solve the IVP with initial condition $y(0) = 1$ by using

(i) Euler's method.
(ii) Modified Euler's method.

(10 marks)

(c) Given the boundary-value problem (BVP)

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + x = 0, \qquad 0 \le x \le 2$$

with conditions $y(0) = 0$ and $y(2) = 1$. Solve the BVP by using finite-difference method by taking $\Delta x = 0.5$.

(8 marks)

- END OF QUESTION -

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FORMULAS

Nonlinear equations

Secant method: $$x_{i+2} = \frac{x_i f(x_{i+1}) - x_{i+1} f(x_i)}{f(x_{i+1}) - f(x_i)}, \quad i = 0, 1, \ldots, n$$

System of linear equations

Gauss-Seidel iteration method: $$x_i^{(k+1)} = \frac{b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)}}{a_{ii}}, \quad i = 1, 2, \ldots, n$$

Thomas Algorithm:

i	1	2	…	n
d_i				
e_i				
c_i				
b_i				
$\alpha_1 = d_1$ $\alpha_i = d_i - c_i \beta_{i-1}$				
$\beta_i = \frac{e_i}{\alpha_i}$				
$y_1 = \frac{b_1}{\alpha_1}$ $y_i = \frac{b_i - c_i y_{i-1}}{\alpha_i}$				
$x_n = y_n$ $x_i = y_i - \beta_i x_{i+1}$				

Interpolation

Newton divided difference:

$$P_n(x) = f_0^{[0]} + f_0^{[1]}(x - x_0) + f_0^{[2]}(x - x_0)(x - x_1) + \ldots + f_0^{[n]}(x - x_0)(x - x_1)\ldots(x - x_{n-1})$$

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Numerical differentiation and integration

Second derivative:

5-point difference: $$f''(x) \approx \frac{-f(x+2h)+16f(x+h)-30f(x)+16f(x-h)-f(x-2h)}{12h^2}$$

Integration:

$\frac{1}{3}$ Simpson's rule: $$\int_a^b f(x)dx \approx \frac{h}{3}\left[f_0 + f_n + 4\sum_{\substack{i=1 \\ i \text{ odd}}}^{n-1} f_i + 2\sum_{\substack{i=2 \\ i \text{ even}}}^{n-2} f_i \right]$$

Eigenvalue

Power Method: $$v^{(k+1)} = \frac{1}{m_{k+1}} A v^{(k)}, \quad k = 0,1,2,\ldots$$

Ordinary differential equations

Initial value problems:

Euler's method: $y_{i+1} = y_i + hy_i'$

Modified Euler's method: $y_{i+1} = y_i + \frac{1}{2}k_1 + \frac{1}{2}k_2$

where $k_1 = hf(x_i, y_i)$, $k_2 = hf(x_i + h, \ y_i + k_1)$

Boundary value problems:

Finite-difference method:

$$y_i' \approx \frac{y_{i+1} - y_{i-1}}{2h}, \qquad y_i'' \approx \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$$

Partial differential equation

Heat Equation: Finite difference method

$$\left(\frac{\partial u}{\partial t}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \quad \Rightarrow \quad \frac{u_{i,j+1} - u_{i,j}}{k} = c^2 \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$