UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2012/2013

COURSE NAME	:	MATHEMATICS FOR ENGINEERING TECHNOLOGY I
COURSE CODE	:	BWM 12203
PROGRAMME	:	1 BNB/1 BND/ 1 BNH/ 1BNK/ 1 BNL/ 1 BNN
EXAMINATION DATE	:	DECEMBER 2012/JANUARY 2012
DURATION	:	2 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS EXAMINATION PAPER CONSISTS OF **SIX (6)** PAGES

Q1 (a) Given

$$f(x)=\begin{cases} x-2 & 0<x\le 2, \\ 2x-4 & 2\le x<4, \\ x^2 & x\ge 4. \end{cases}$$

(i) Find $\lim_{x\to 0^+} f(x)$, $\lim_{x\to 2^-} f(x)$, $\lim_{x\to 2^+} f(x)$ and $\lim_{x\to 4^-} f(x)$

(ii) Check whether the function is continuous at the point $x=2$.

(6 marks)

(b) Evaluate the following limits.

(i) $\lim_{x\to 0}\left[\dfrac{e^x(x^2-2x+1)}{(x-1)}\right]$

(ii) $\lim_{x\to 2b}\dfrac{(x+b)^3}{x^2-b^2}$

(6 marks)

(c) Use L'Hopital's rule to find the limit below.

(i) $\lim_{x\to 0^+}\left[\dfrac{1}{\ln(2x+1)}-\dfrac{1}{2x}\right]$

(ii) $\lim_{x\to 0}\dfrac{\ln x}{\csc x}$

(13 marks)

Q2 (a) (i) If $x^2\cos y+x=y$, find $\dfrac{dy}{dx}$ in terms of x and y.

(ii) If $y=\dfrac{x}{x^2+2}$, find $\dfrac{dy}{dx}$, $\dfrac{d^2y}{dx^2}$ and give your answers in the simplest form.

(10 marks)

(b) Evaluate the following integrals by using any suitable integration techniques or the given respective techniques.

(i) $\int e^{\cos x} \sin x \, dx$

(ii) $\int_0^{\pi} e^x \cos x \, dx$, integration by parts method.

(iii) $\int \frac{6x+7}{x^2+4x+4} dx$

(11 marks)

(c) Use integration to find the area of the region in the xy-plane bounded by the curve $y = \cos x$ and $y = \sin x$ for $0 \le x \le \frac{\pi}{4}$.

(4 marks)

Q3 (a) Show that the Maclaurin series for $f(x) = \frac{1}{1-x}$ is as follows:

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots$$

Hence, find the Maclaurin series for the function:

$$g(x) = \frac{1+x}{1-x}$$

(7 marks)

(b) Use any convergence test to determine whether the following series converges or diverges.

(i) $\sum_{n=2}^{\infty} \frac{(n+2)!}{(n-1)!\,2^n}$

(ii) $\sum_{n=1}^{\infty} \left(\frac{1}{n+3} - \frac{1}{n+4} \right)$

(11 marks)

(c) Find the first 4 terms of the Maclaurin series for the function $f(x)=\cos(x^2)$. Hence, use this series to estimate the value of the definite integral

$$\int_0^1 \cos(x^2)\,dx$$

(7 marks)

Q4 Given a helix, $\underline{r}(t)=3(\cos t)\underline{i}+3(\sin t)\underline{j}+t\underline{k},\ \ 0\le t\le 4\pi$.

(a) Sketch the helix. (6 marks)

(b) Find the unit tangent vector. (6 marks)

(c) Find the curvature for $\underline{r}(t)$. (6 marks)

(d) Find the arc length for $\underline{r}(t)$. (7 marks)

- END OF QUESTIONS -

DIFFERENTIATION AND INTEGRATION FORMULA

Differentiation	Integration	Integration Involving Radical Function
$\frac{d}{dx}\frac{x^{n+1}}{n+1} = x^n, \quad n \neq -1$	$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$	$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + C$
$\frac{d}{dx}\ln x = \frac{1}{x}$	$\int \frac{1}{x} dx = \ln x + C$	$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + C$
$\frac{d}{dx}\sin x = \cos x$	$\int \cos x \, dx = \sin x + C$	$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
$\frac{d}{dx}\cos x = -\sin x$	$\int \sin x \, dx = -\cos x + C$	$\int \frac{-1}{a^2 + x^2} dx = \frac{1}{a}\cot^{-1}\left(\frac{x}{a}\right) + C$
$\frac{d}{dx}\tan x = \sec^2 x$	$\int \sec^2 x \, dx = \tan x + C$	$\int \frac{1}{\lvert x\rvert\sqrt{x^2 - a^2}} dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C$
$\frac{d}{dx}\cot x = -\csc^2 x$	$\int \csc^2 x \, dx = -\cot x + C$	$\int \frac{-1}{\lvert x\rvert\sqrt{x^2 - a^2}} dx = \frac{1}{a}\csc^{-1}\left(\frac{x}{a}\right) + C$
$\frac{d}{dx}\sec x = \sec x \tan x$	$\int \sec x \tan x \, dx = \tan x + C$	
$\frac{d}{dx}\csc x = -\csc x \cot x$	$\int \csc x \cot x \, dx = -\csc x + C$	
$\frac{d}{dx}e^x = e^x$	$\int e^x \, dx = e^x + C$	

IDENTITIES OF TRIGONOMETRY

Trigonometry		Substitutions: Expression	Substitutions: Trigonometric
$\cos^2 x + \sin^2 x = 1$	$\sin 2x = 2\sin x \cos x$	$\sqrt{x^2 + k^2}$	$x = k\tan\theta$
$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$	$1 + \tan^2 x = \sec^2 x$	$\sqrt{x^2 - k^2}$	$x = k\sec\theta$
$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$	$1 + \cot^2 x = \csc^2 x$	$\sqrt{k^2 - x^2}$	$x = k\sec\theta$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$	$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$		
$\cos ax \cos bx = \frac{1}{2}[\cos(a+b)x + \cos(a-b)x]$	$\cos 2x = \cos^2 x - \sin^2 x$		
$\sin ax \sin bx = \frac{1}{2}[-\cos(a+b)x + \cos(a-b)x]$	$= 2\cos^2 x - 1$		
$\sin ax \cos bx = \frac{1}{2}[\sin(a+b)x + \sin(a-b)x]$	$= 1 - 2\sin^2 x$		

Curvature		**Length of Curve and Area of a Surface of Revolution**	
$y = f(x)$	$y = f(t)$	$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$	$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$
$\kappa = \dfrac{\left\lvert \frac{d^2 y}{dx^2} \right\rvert}{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}$	$\kappa = \dfrac{\lvert \dot{x}\ddot{y} - \dot{y}\ddot{x} \rvert}{\left[\dot{x}^2 + \dot{y}^2\right]^{\frac{3}{2}}}$	$L = 2\pi \int_{x_1}^{x_2} y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$	$L = 2\pi \int_{x_1}^{x_2} x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$
		$L = 2\pi \int_{t_1}^{t_2} y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$	

FINAL EXAMINATION		
SEMESTER / SESSION: SEM I / 2012/2013	PROGRAMME:	1 BDD 2 BDD/BED/BEH/ BFF 3 BDD/ BEE/BFF 4 BDD/ BEE/BFF
COURSE NAME : MATHEMATICS FOR ENGINEERING TECHNOLOGY I	COURSE CODE:	BWM 12203