UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2012/2013

COURSE NAME	:	ENGINEERING MATHEMATICS II
COURSE CODE	:	BDA 14103 / BWM 10203/ BSM 1923
PROGRAMME	:	1 BDD/2 BDD/4 BDD/4 BFF
EXAMINATION DATE	:	JANUARY 2013
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS IN **PART A** AND **THREE (3)** QUESTIONS IN **PART B**

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

PART A

Q1 A periodic function $f(x)$ is defined by

$$f(x)=\begin{cases}1, & -1<x<0,\\ 4, & 0<x<1.\end{cases}$$

and

$$f(x)=f(x+2).$$

(a) Sketch the graph of $f(x)$ over $-3<x<3$.

(3 marks)

(b) Calculate the Fourier coefficients, a_0 and a_n.

(8 marks)

(c) Show that the Fourier coefficient, b_n corresponding to $f(x)$ is given by $b_n=\frac{3}{n\pi}[1-\cos n\pi]$.

(6 marks)

(d) Hence, obtain the corresponding Fourier series of $f(x)$.

(3 marks)

Q2 A uniform string of length π is fastened at both of its ends $x=0$ and $x=\pi$. The string is oscillating with initial velocity given as $\frac{\delta u}{\delta t}(x,0)=g(x)=0$ and the initial state of the string is given by $u(x,0)=f(x)=2+x,\quad 0<x<\pi$.

This is a wave problem given by,

$$\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}, \qquad 0<x<\pi, \quad t>0.$$

The series solution of this problem is,

$$u(x,t)=\sum_{n=1}^{\infty}\left\{a_n\cos\left(\frac{n\pi ct}{l}\right)+b_n\sin\left(\frac{n\pi ct}{l}\right)\right\}\sin\left(\frac{n\pi x}{l}\right)$$

(a) Write

(i) the value of the physical constant c,
(ii) the initial conditions,
(iii) the boundary conditions of the given problem.

(8 marks)

(b) By using

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin(nx)\,dx \text{, and } b_n = \frac{2}{n\pi c}\int_0^{\pi} g(x)\sin(nx)\,dx$$

determine a_n and b_n .

(10 marks)

(c) Hence, write the particular solution $u(x,t)$.

(2 marks)

PART B

Q3 (a) Given $y = e^x \sin x$. Show that $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$.

(4 marks)

(b) Solve the differential equation by using the method of integrating factor.

$$x\frac{dy}{dx} - y = x$$

(4 marks)

(c) By using a substitution of $y = vx$, and $\frac{dy}{dx} = v + x\frac{dv}{dx}$, find the solution of

$$y' = \frac{x^2 + y^2}{xy}, \text{ given } y(1) = 1.$$

(6 marks)

(d) Solve the following first order differential equation by using the appropriate method.

$$(x + \sin y)dx + (x\cos y - 2y)dy = 0.$$

(6 marks)

Q4 (a) Find the particular solution for the second order differential equation

$$y'' + 2y' - 8y = 0, \qquad y(0) = 5, \qquad y'(0) = -12.$$

(6 marks)

(b) Find the general solution for the second order differential equation

$$y'' + 2y' + 5y = x^2 - 1.$$

by using the undetermined coefficient method. (7 marks)

(c) Given a non-homogeneous second order differential equation

$$y'' - 6y' + 9y = xe^{3x}.$$

Find the general solution for the equation by using variation of parameters method.

(7 marks)

Q5 (a) Find the following transforms.

(i) $L\{(1-e^{-t})^2\}$,

(ii) $L^{-1}\left\{\dfrac{s+3}{s^2+8s+12}\right\}$.

(12 marks)

(b) Consider the function

$$f(t) = \begin{cases} e^t, & 0 \le t < 2, \\ t-2, & t \ge 2. \end{cases}$$

(i) Write the function $f(x)$ in the form of unit step function.

(ii) Then, find the Laplace transform of $f(t)$.

(8 marks)

Q6 (a) Given $f(t)=\begin{cases}1, & 0\le t<3,\\ 2, & 3\le t<5,\\ -1, & t\ge 5.\end{cases}$

(i) Sketch the graph of $f(t)$.

(ii) Write $f(t)$ in unit step function.

(ii) Then, find the Laplace Transform of $f(t)$.

(12 marks)

(b) By using Laplace transform, solve

$$y''-6y'+8y=2, \qquad y(0)=0 \text{ and } y'(0)=0.$$

(8 marks)

- END OF QUESTION -

FINAL EXAMINATION

SEMESTER / SESSION: SEM 1 / 2012/2013 COURSE : 1 BDD / 2 BDD / 4 BDD 4 BFF

SUBJECT : ENGINEERING MATHEMATICS II CODE : BDA 14103 / BWM 10203/ BSM 1923

FORMULA

Second-order Differential Equation

The roots of characteristic equation and the general solution for differential equation $ay'' + by' + cy = 0$.

Characteristic equation: $am^2 + bm + c = 0$.		
Case	The roots of characteristic equation	General solution
1.	Real and different roots: m_1 and m_2	$y = Ae^{m_1 x} + Be^{m_2 x}$
2.	Real and equal roots: $m = m_1 = m_2$	$y = (A + Bx)e^{mx}$
3.	Complex roots: $m_1 = \alpha + \beta i$, $m_2 = \alpha - \beta i$	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

The method of undetermined coefficients

For non-homogeneous second order differential equation $ay'' + by' + cy = f(x)$, the particular solution is given by $y_p(x)$:

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + A_{n-1}x^{n-1} + \cdots + A_1 x + A_0$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(P\cos\beta x + Q\sin\beta x)$
$P_n(x)e^{\alpha x}$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)e^{\alpha x}$
$P_n(x)\begin{cases}\cos\beta x\\ \sin\beta x\end{cases}$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)\cos\beta x +$ $x^r(C_n x^n + C_{n-1}x^{n-1} + \cdots + C_1 x + C_0)\sin\beta x$
$Ce^{\alpha x}\begin{cases}\cos\beta x\\ \sin\beta x\end{cases}$	$x^r e^{\alpha x}(P\cos\beta x + Q\sin\beta x)$
$P_n(x)e^{\alpha x}\begin{cases}\cos\beta x\\ \sin\beta x\end{cases}$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)e^{\alpha x}\cos\beta x +$ $x^r(C_n x^n + C_{n-1}x^{n-1} + \cdots + C_1 x + C_0)e^{\alpha x}\sin\beta x$

Note : r is the least non-negative integer (r = 0, 1, or 2) which determine such that there is no terms in particular integral $y_p(x)$ corresponds to the complementary function $y_c(x)$.

The method of variation of parameters

If the solution of the homogeneous equation $ay'' + by' + cy = 0$ is $y_c = Ay_1 + By_2$, then the particular solution for $ay'' + by' + cy = f(x)$ is

$$y = uy_1 + vy_2,$$

where $u = -\int \frac{y_2 f(x)}{aW}dx + A$, $v = \int \frac{y_1 f(x)}{aW}dx + B$ and $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_2 y_1'$.

FINAL EXAMINATION

SEMESTER / SESSION: SEM I / 2012/2013

COURSE : 1 BDD / 2 BDD / 4 BDD
4 BFF

SUBJECT : ENGINEERING MATHEMATICS II

CODE : BDA 10103 / BWM 10203 /BSM 1923

Laplace Transform

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
A	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
e^{at}	$\frac{1}{s-a}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
$\sin at$	$\frac{a}{s^2+a^2}$	$\delta(t-a)$	e^{-as}
$\cos at$	$\frac{s}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$\int_0^t f(u)g(t-u)\,du$	$F(s)\cdot G(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$y(t)$	$Y(s)$
$t^n,\ n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	$y'(t)$	$sY(s)-y(0)$
$e^{at}f(t)$	$F(s-a)$	$y''(t)$	$s^2Y(s)-sy(0)-y'(0)$
$t^n f(t),\ n=1,2,3,\ldots$	$(-1)^n \frac{d^n}{ds^n}F(s)$		

Fourier Series

Fourier series expansion of periodic function with period $2L$	Fourier half-range series expansion
$f(x)=\frac{1}{2}a_0+\sum_{n=1}^{\infty}a_n\cos\frac{n\pi x}{L}+\sum_{n=1}^{\infty}b_n\sin\frac{n\pi x}{L}$	$f(x)=\frac{1}{2}a_0+\sum_{n=1}^{\infty}a_n\cos\frac{n\pi x}{L}+\sum_{n=1}^{\infty}b_n\sin\frac{n\pi x}{L}$
where	where
$a_0=\frac{1}{L}\int_{-L}^{L}f(x)\,dx$	$a_0=\frac{2}{L}\int_0^{L}f(x)\,dx$
$a_n=\frac{1}{L}\int_{-L}^{L}f(x)\cos\frac{n\pi x}{L}\,dx$	$a_n=\frac{2}{L}\int_0^{L}f(x)\cos\frac{n\pi x}{L}\,dx$
$b_n=\frac{1}{L}\int_{-L}^{L}f(x)\sin\frac{n\pi x}{L}\,dx$	$b_n=\frac{2}{L}\int_0^{L}f(x)\sin\frac{n\pi x}{L}\,dx$