UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2012/2013

COURSE NAME	:	ENGINEERING MATHEMATICS II
COURSE CODE	:	BWM 11903/ BDU 11003
PROGRAMME	:	1 BDC/BDM 3 BDC
EXAMINATION DATE	:	JUNE 2013
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS IN **PART A** AND **THREE** (3) QUESTIONS IN **PART B.**

THIS EXAMINATION PAPER CONSISTS OF **SIX (6)** PAGES

PART A

Q1 A periodic function $f(x)$ is defined by

$$f(x)=\begin{cases}1 & , -1\le x<0,\\ 1-x, & 0\le x<1,\end{cases}$$

and $$f(x)=f(x+2).$$

(a) Sketch the graph of $f(x)$ over $-3\le x\le 3$.

(2 marks)

(b) Show that the Fourier coefficients corresponding to $f(x)$ are

$$a_0=\frac{3}{2},\ a_n=\begin{cases}\dfrac{2}{n^2\pi^2}, & n \text{ is odd}\\ 0 & , n \text{ is even}\end{cases} \text{ and } b_n=\begin{cases}-\dfrac{1}{n\pi}, & n \text{ is odd}\\ \dfrac{1}{n\pi} & , n \text{ is even}\end{cases}.$$

(15 marks)

(c) Write the Fourier series of $f(x)$ by giving your answers for the first three nonzero terms of a_n and b_n.

(3 marks)

Q2 A uniform string of length π is fastened at both of its ends $x=0$ and $x=\pi$. The string is set oscillating with initial velocity given as

$$g(x)=x,\ 0\le x\le\pi$$

and initial state of the string is defined by

$$f(x)=x(2-x),\ 0\le x\le\pi.$$

By applying the wave equation, $u_{tt}=4u_{xx}$, the subsequent displacement of the string is

$$u(x,t)=\sum_{n=1}^{\infty}\left[A_n\cos 2nt+B_n\sin 2nt\right]\sin nx.$$

(a) Write down

(i) the initial conditions and

(2 marks)

(ii) the boundary conditions of the given problem.

(2 marks)

(b) Using the fact that

$$A_n=\frac{2}{\pi}\int_0^{\pi}f(x)\sin nx\,dx \text{ and } B_n=\frac{1}{n\pi}\int_0^{\pi}g(x)\sin nx\,dx,$$

find A_n and B_n.

(16 marks)

PART B

Q3 (a) Show that

(i) $$\left(2x + y\cos(xy)\right)dx + \left(x\cos(xy)\right)dy = 0$$

is an exact differential equation and then find its solution.

(ii) $$xdy - ydx - \sqrt{x^2 - y^2}\,dx = 0$$

is homogeneous and then find its solution.

(14 marks)

(b) The total population of Malaysia, according to *Census 2000*, was 23.27 million in the year 2000 compared to 18.38 million in the year 1991. *Census 2000* revealed that an average annual population growth rate over the 1991 – 2000 periods was similar to that of the 1980 – 1991. Find the number of people living in Malaysia in 1980. Assume that the rate of population growth, $y'(t)$ is related to the population size, $y(t)$ by the following differential equation

$$y'(t) = ky(t)$$

where k is a constant.

(Hint: assume that $t = 0$ in the year 1980)

(6 marks)

Q4 (a) Solve the differential equation

$$y'' - y' = x.$$

(8 marks)

(b) Find the general solution of

$$y'' - 4y' + 4y = \frac{2e^{2x}}{x}.$$

(12 marks)

Q5 (a) Find the Laplace transform for each of the following function:

(i) $f(t) = [1 - 2\cos t]^2$.

(ii) $f(t) = (e^t + t)\cosh(\frac{1}{2}t)$.

(iii) $f(t) = \delta(t - \frac{1}{2}) - t^4\delta(t - 2)$.

(13 marks)

(b) The unit step function is defined by

$$H(t-a) = \begin{cases} 0, & 0 \le t < a \\ 1, & t \ge a. \end{cases}$$

(i) Sketch the function $f(t) = t\ H(t-3)$.

(ii) Find the Laplace transform for $f(t) = t\ H(t-3)$.

(7 marks)

Q6 (a) Use the convolution theorem to find the inverse Laplace transform for

$$\frac{1}{s^2(s^2+1)}.$$

(8 marks)

(b) Solve the boundary value problem

$$y'' + y = \delta(t - \pi) + 2t,\ y(0) = 0,\ y(\tfrac{\pi}{2}) = \pi.$$

(Hint: Use the result obtained in **Question 6(a)**).

(12 marks)

-END OF QUESTION-

FINAL EXAMINATION

Formulae

Characteristic Equation and General Solution

Case	Roots of the Characteristic Equation	General Solution
1	m_1 and m_2 ; real and distinct	$y = Ae^{m_1 x} + Be^{m_2 x}$
2	$m_1 = m_2 = m$; real and equal	$y = (A + Bx)e^{mx}$
3	$m = \alpha \pm i\beta$; imaginary	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Undetermined Coefficients

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + \cdots + A_1 x + A_0$	$x^r(B_n x^n + \cdots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(p\cos\beta x + q\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Variation of Parameters

Wronskian	Parameter	Solution
$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$	$u_1 = -\int \frac{y_2 f(x)}{W} dx, \quad u_2 = \int \frac{y_1 f(x)}{W} dx$	$y_p = u_1 y_1 + u_2 y_2$