UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2012/2013

COURSE NAME	:	ENGINEERING MATHEMATICS III
COURSE CODE	:	BWM 20403/ BSM 2913
PROGRAMME	:	1 BEF/1BEU/ 2 BEB / 2 BEF/ 2 BDD/ 2 BEC/ 3 BEF / 3 BFF / 3 BDD/ 3 BEH / 4 BDD/ 4 BEE / 4 BFF
EXAMINATION DATE	:	JUNE 2013
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS EXAMINATION PAPER CONSISTS OF **SIX (6)** PAGES

Q1 (a) Let $z = x^2 + y^2$ where $x = \frac{1}{t}$ and $y = \ln t$. Find $\frac{dz}{dt}$ in two ways:

(i) by first expressing z explicitly in terms of t.

(ii) by using chain rule.

(6 marks)

(b) If y is a differentiable function of x such that $x^3 + 4x^2y - 3xy + y^2 = 0$, find $\frac{dy}{dx}$.

(5 marks)

(c) Find the local extrema and saddle point(s) of $f(x, y) = e^{-\frac{1}{3}x^3 + x - y^2}$

(9 marks)

Q2 (a) Use polar coordinates to evaluate $\iint_R \sqrt{x^2 + y^2}\, dA$, where R is the region inside the circle $(x-1)^2 + y^2 = 1$ in the first quadrant.

(7 marks)

(b) Consider an object which is bounded above by the inverted paraboloid $z = 16 - x^2 - y^2$ and below by the xy-plane. Suppose that the density of the object is given by $\delta(x, y, z) = 8 + x + y$. Find the mass of the object by using cylindrical coordinates.

(6 marks)

(c) Use spherical coordinates to evaluate $\int_{-\sqrt{2}}^{0} \int_{0}^{\sqrt{2-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} \sqrt{x^2 + y^2 + z^2}\, dz\, dx\, dy$

(9 marks)

Q3 (a) Sketch the graph for $r(t) = 5\sin t\,\mathbf{i} - 2\cos t\,\mathbf{j}$, $t \in \boldsymbol{R}$ vector-valued function.

(3 marks)

(b) Find a vector-valued function that represents the curve of intersection of the cylinder $x^2 + y^2 = 8$ and the plane $y + z = 1$.

(5 marks)

(c) Find the curvature of the line $\mathbf{r}(s) = (2-6s)\,\mathbf{i} + 3s\,\mathbf{j}$.

(5 marks)

(d) Find the velocity, speed and acceleration of the particle at $t = \pi$ with the position vector $\mathbf{r}(t) = t\cos t\,\mathbf{i} + e^{t}\sin t\,\mathbf{j} + 2t\,\mathbf{k}$.

(7 marks)

Q4 (a) Use Green's Theorem to evaluate the line integral $\oint_C (x+y^2)\,dx + 2x\,dy$ where C is the boundary of the region between $x^2+y^2=1$ and $x^2+y^2=9$.

(10 marks)

(b) Find the work done by force field $\mathbf{F}(x,y) = x\,\mathbf{i} + (2x+y)\,\mathbf{j}$ along the curve C, where C is the upper semicircle that starts from (1,0) and ends at (0,1).

(10 marks)

Q5 (a) Let σ be the surface of the plane $z = xy$ with $0 \le x \le 1$, $0 \le y \le 2$ and $\mathbf{F}(x,y,z) = z^2\,\mathbf{j} - x^3y^4\,\mathbf{k}$ across σ. Assume that the $\mathbf{n}$ is the unit normal vector oriented outward. Evaluate $\iint_{\sigma} \mathbf{F}.\mathbf{n}\,dS$.

(10 marks)

(b) Let σ be the surface of the solid G enclosed by upper hemisphere $z = \sqrt{9-x^2-y^2}$ and plane $z = 0$, oriented outward. Use divergence theorem to find the flux of vector field $\mathbf{F}(x,y,z) = \frac{1}{3}x^3\,\mathbf{i} + \frac{1}{3}y^3\,\mathbf{j} + \frac{1}{3}z^3\,\mathbf{k}$ across σ.

(10 marks)

- END OF QUESTION –

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Formulae

Polar coordinate: $x = r\cos\theta$, $y = r\sin\theta$, $\theta = \tan^{-1}(y/x)$, and $\iint\limits_R f(x,y)dA = \iint\limits_R f(r,\theta)\, r\, dr\, d\theta$

Cylindrical coordinate: $x = r\cos\theta$, $y = r\sin\theta$, $z = z$, $\iiint\limits_G f(x,y,z)dV = \iiint\limits_G f(r,\theta,z)\, r\, dz\, dr\, d\theta$

Spherical coordinate: $x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$, $z = \rho\cos\phi$, $x^2+y^2+z^2=\rho^2$, $0\le\theta\le 2\pi$, $0\le\phi\le\pi$, and

$$\iiint\limits_G f(x,y,z)dV = \iiint\limits_G f(\rho,\phi,\theta)\rho^2\sin\phi\, d\rho\, d\phi\, d\theta$$

Directional derivative: $D_{\mathbf{u}}f(x,y) = (f_x\mathbf{i} + f_y\mathbf{j})\cdot\mathbf{u}$

Let $\mathbf{F}(x,y,z) = M\,\mathbf{i} + N\,\mathbf{j} + P\,\mathbf{k}$ is vector field, then

the **divergence** of $\mathbf{F} = \nabla\cdot\mathbf{F} = \dfrac{\partial M}{\partial x} + \dfrac{\partial N}{\partial y} + \dfrac{\partial P}{\partial z}$

the **curl** of $\mathbf{F} = \nabla\times\mathbf{F} = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z}\\ M & N & P\end{vmatrix} = \left(\dfrac{\partial P}{\partial y} - \dfrac{\partial N}{\partial z}\right)\mathbf{i} - \left(\dfrac{\partial P}{\partial x} - \dfrac{\partial M}{\partial z}\right)\mathbf{j} + \left(\dfrac{\partial N}{\partial x} - \dfrac{\partial M}{\partial y}\right)\mathbf{k}$

Let C is a smooth curve given by $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, t is parameter, then

the unit tangent vector: $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

the unit normal vector: $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

the binormal vector: $\mathbf{B}(t) = \mathbf{T}(t)\times\mathbf{N}(t)$

the curvature: $\kappa = \dfrac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|} = \dfrac{\|\mathbf{r}'(t)\times\mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$

the radius of curvature: $\rho = 1/\kappa$

Green Theorem: $\oint\limits_C M\,dx + N\,dy = \iint\limits_R \left(\dfrac{\partial N}{\partial x} - \dfrac{\partial M}{\partial y}\right)dA$

Gauss Theorem: $\iint\limits_S \mathbf{F}\bullet\mathbf{n}\,dS = \iiint\limits_G \nabla\bullet\mathbf{F}\,dV$

Stokes' Theorem: $\oint\limits_C \mathbf{F}\bullet d\mathbf{r} = \iint\limits_S (\nabla\times\mathbf{F})\bullet\mathbf{n}\,dS$

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Flux Integrals: $\iint_\sigma \mathbf{F} \bullet \mathbf{n}\, dS$

i) oriented upward, $\mathbf{n} = \frac{\nabla\phi}{|\nabla\phi|} = \frac{-\frac{\partial z}{\partial x}\mathbf{i} - \frac{\partial z}{\partial y}\mathbf{j} + \mathbf{k}}{\sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}}$

ii) oriented downward, $\mathbf{n} = \frac{\nabla\phi}{|\nabla\phi|} = \frac{\frac{\partial z}{\partial x}\mathbf{i} + \frac{\partial z}{\partial y}\mathbf{j} - \mathbf{k}}{\sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}}$

Arc length

If $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$, $t \in [a,b]$, then the **arc length** $s = \int_a^b \|\mathbf{r}'(t)\| dt = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2}\, dt$

If $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, $t \in [a,b]$, then the **arc length** $s = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}\, dt$

Tangent Plane

$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$

Extreme of two variable functions

$G(x, y) = f_{xx}(x, y) f_{yy}(x, y) - (f_{xy}(x, y))^2$

Case1: If $G(a,b) > 0$ and $f_{xx}(x, y) < 0$ then f has local maximum at (a,b)

Case2: If $G(a,b) > 0$ and $f_{xx}(x, y) > 0$ then f has local minimum at (a,b)

Case3: If $G(a,b) < 0$ then f has a saddle point at (a,b)

Case4: If $G(a,b) = 0$ then no conclusion can be made.

In 2-D: Lamina

Mass: $m = \iint_R \delta(x, y)\, dA$, where $\delta(x, y)$ is a density of lamina.

Moment of mass: (i) about y-axis, $M_y = \iint_R x\delta(x, y)\, dA$, (ii) about x-axis, $M_x = \iint_R y\delta(x, y)\, dA$

Centre of mass, $(\bar{x}, \bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m}\right)$

Moment inertia: (i) $I_y = \iint_R x^2\delta(x, y)\, dA$, (ii) $I_x = \iint_R y^2\delta(x, y)\, dA$, (iii) $I_o = \iint_R (x^2 + y^2)\delta(x, y)\, dA$

In 3-D: Solid

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Mass, $m = \iiint_G \delta(x,y,z)\,dV$. If $\delta(x,y,z) = c$, c is a constant, then $m = \iiint_G dA$ is volume.

Moment of mass

(i) about yz-plane, $M_{yz} = \iiint_G x\delta(x,y,z)\,dV$

(ii) about xz-plane, $M_{xz} = \iiint_G y\delta(x,y,z)\,dV$

(iii) about xy-pane, $M_{xy} = \iiint z\delta(x,y,z)\,dV$

Centre of gravity, $(\bar{x},\bar{y},\bar{z}) = \left(\frac{M_{yz}}{m},\frac{M_{xz}}{m},\frac{M_{xy}}{m}\right)$

Moment inertia

(i) about x-axis: $I_x = \iiint_G (y^2+z^2)\delta(x,y,z)\,dV$

(ii) about y-axis: $I_y = \iiint_G (x^2+z^2)\delta(x,y,z)\,dV$

(iii) about z-axis: $I_z = \iiint_G (x^2+y^2)\delta(x,y,z)\,dV$