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Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2013/2014

COURSE NAME	:	ENGINEERING MATHEMATICS IV
COURSE CODE	:	BWM30603/BSM3913
PROGRAMME	:	3 BDD
EXAMINATION DATE	:	DECEMBER 2013/JANUARY 2014
DURATION	:	3 HOURS
INSTRUCTION	:	A) ANSWER **ALL** QUESTIONS IN SECTION A B) ANSWER **TWO (2)** QUESTIONS IN SECTION B

THIS QUESTION PAPER CONSISTS OF **TWELVE (12)** PAGES

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SECTION A

Q1 (a) A new conductor material is under examination in order to inspect the heat transfer performance. The material is fully insulated so that the heat transfer is only one dimensional in axial direction (x-axis). The initial temperature of the material is at room temperature of 25 °C. At one end (point A) is heated while another end (point E) is attached to a cooler system, as shown in **FIGURE Q1 (a)**.

The unsteady state heating equation follows a heat equation

$$\frac{\partial T}{\partial t} - K\frac{\partial^2 T}{\partial x^2} = 0,$$

where K is thermal diffusivity of steel material and x is the longitudinal coordinate of the bar. The thermal diffusivity of the steel is given as $K = 8\ \text{mm}^2/\text{s}$, $\Delta x = h = 10$ and $\Delta t = k = 4$.

(i) Propose a heat equation in explicit finite-difference form to investigate the temperature of point A, B, C, D and E for every 4 seconds.

(3 marks)

(ii) Draw finite difference grid to predict the temperature of point A, B, C, D and E up to 8 seconds. Specify all known temperatures in the grid.

(2 marks)

(iii) Determine the temperatures of point A, B, C, D and E at 8 seconds.

(5 marks)

(b) A 2-meter metal wire with a density of 5 kg/m is stretched with a tension 50 N and hold on its ends in a rigid holder so that the string is straight (gravity is not considered) and there is no initial displacement, as shown in **FIGURE Q1 (b)**. The string has 5 assessment points (evenly distributed). The string is then disturbed by velocity to vibrate the spring. This initial velocity disturbance to all points vibrates the string. The free vibration of a string follows the equation

$$\frac{\partial^2 y}{\partial^2 t} = \left(\frac{T}{\rho}\right)\frac{\partial^2 y}{\partial^2 x}.$$

(i) Propose finite-difference formulations for wave equation and illustrate this problem into a molecule graph.

(3 marks)

(ii) Illustrate the analysis of grid to determine the displacements of all assessment points (A, B, C, D, E), to analyze the displacement of the string at $t = 0$ s, $t = 0.01$ s and $t = 0.02$ s. Include all boundary and initial conditions, with finite-difference approximation for wave equation.

(6 marks)

(iii) Determine the displacements of all points at $t = 0.01$ s and $t = 0.02$ s.

(6 marks)

Q2 A bar structure is supported by a linear axial spring. The structure is indirectly loaded by a lever system in **FIGURE Q2**. The loading in one end of the lever is 2000 N.

(a) Draw a finite element representative model consists of three axial elements and three nodes. You have to indicate clearly the elements, the nodes, the constraints and the loadings.

(3 marks)

(b) Calculate the stiffness matrix and the force vector of each element.

(12 marks)

(c) Write the structural global stiffness matrix and the global force vector.

(5 marks)

(d) Determine the displacements of node 2 and node 3 as shown in **FIGURE Q2**.

(5 marks)

SECTION B

Q3 (a) Find the **largest positive root** of the equation

$$x^2 + 4\sin(2x) - 2 = 0, \quad \text{for } 0 \le x \le 3$$

by using bisection method. Iterate until $f(c_i) < 0.005$.

(10 marks)

(b) A thin rod is positioned between two walls which are held at constant temperatures. Heat flows through the rod as well as between the rod and the surrounding air. A differential equation based on heat conversion for the steady-state case can be transformed into the following equation

$$-\left[\left(\frac{T_{i+1} - 2T_i + T_{i-1}}{(\Delta x)^2}\right) + h_f(T_a - T_i)\right] = 0,$$

where T_i is temperature at node i ($^\circ$C), x is distance along the rod (m), h_f is a heat transfer coefficient between the rod and the ambient air (m^{-2}) and T_a is the temperature of the surrounding air ($^\circ$C). The parameter values for the heat problem are $T_a = 20$, $h_f = 0.02$, $\Delta x = 25$, $T(x=0) = 40$ and $T(x=100) = 200$.

(i) Express the above heated rod problem into an algebraic equation.

(5 marks)

(ii) Hence, solve the system by using Gauss-Seidel iteration method with initial temperatures $T_1 = 66$, $T_2 = 96$ and $T_3 = 142$.

(10 marks)

Q4 (a) The vapour pressure, P (mm Hg) of water is presented as a function of temperature, T (°C) as listed in **TABLE Q4 (a)**.

TABLE Q4 (a)

T	40	48	56	64	72	80
P	55	a	124	170	b	260

Find the values of a and b by using Lagrange interpolating polynomial.

(10 marks)

(b) The position of a particle (in distance unit) along a straight line is given by the following equation,

$$s(t) = 1.5t^3 - 13.5t^2 + 22.5t,$$

where t is measured in seconds. Determine the velocity and acceleration of the particle at $t = 6$ sec. by using 3-point central difference formula with any small step length.

(4 marks)

(c) **FIGURE Q4 (c)** shows the graph of ellipse, where a is the major axis and b is the minor axis. The perimeter, P of the ellipse is given by

$$P = 4a \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 \theta} \, d\theta,$$

where $k = \dfrac{\sqrt{a^2 - b^2}}{a}$. Calculate P with $n = 10$ by using suitable Simpson's rule.

(11 marks)

Q5 (a) Given

$$A = \begin{pmatrix} 5 & 0 & 0 \\ 1 & 1 & -1 \\ 0 & 3 & 5 \end{pmatrix}.$$

By taking $v^{(0)} = \begin{pmatrix} 0 & 1 & -0.5 \end{pmatrix}^T$, calculate the smallest eigenvalue and its corresponding eigenvector by using shifted power method, if $\lambda_{\text{largest}} = 5$. Iterate until $|m_{k+1} - m_k| < 0.005$.

(5 marks)

(b) A mass balance for a chemical in a complete mixed reactor can be written as an initial-value problem (IVP) as follows

$$V\frac{dc}{dt} + Qc + kVc^2 = F, \quad c(0) = 2,$$

where V is volume (m^3), c is concentration (g/m^3), F is feed rate (g/min), Q is flow rate (m^3/min) and k is a second order reaction rate $(\text{m}^3/\text{g/min})$. Given that the parameter values are $V = 12$, $F = 174$, $Q = 3$, $k = 0.15$ and $V = 12$. Solve the IVP for $0 \le t \le 0.4$ with step length of $h = 0.2$ by using

(i) Euler method.

(4 marks)

(ii) modified Euler method.

(4 marks)

(c) **FIGURE Q5 (c)** shows a thin rod of length, l that moving in the xy-plane. The rod is fixed with a pin on one end and a mass at the other end. This system is represented in the form of boundary-value problem (BVP) as follows

$$\theta''(t) - \frac{g}{l}\theta(t) = 0, \text{ for } 0 \le t \le 0.3,$$

where boundary conditions are $\theta(0) = 0$ and $\theta'(0.3) = 1$. The parameter values are given as $g =$ gravitational force (9.81 m/s^2) and $l = 0.9$ m. Approximate the angle θ (in radian) for $h = 0.1$ by using finite-difference method.

(12 marks)

- END OF QUESTION -

FINAL EXAMINATION

SEMESTER/SESSION: SEM I/2013/2014 PROGRAMME : 3 BDD
COURSE NAME : ENGINEERING MATHEMATICS IV COURSE CODE: BWM30603/BSM3913

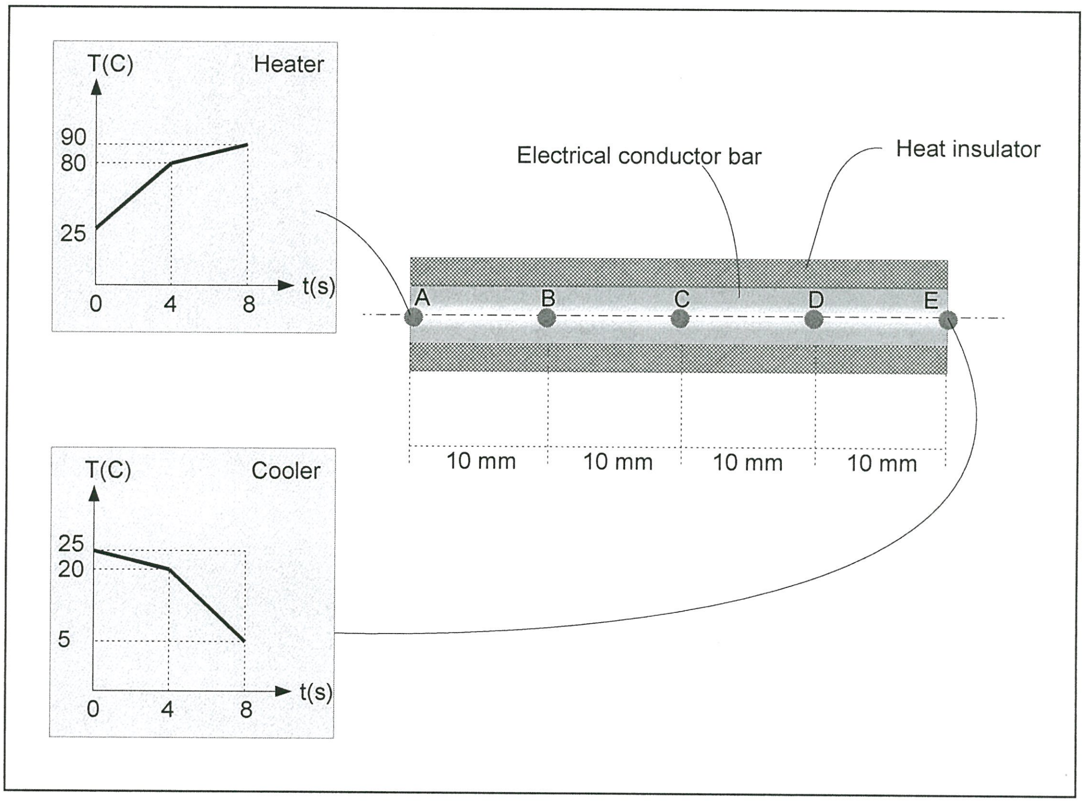

FIGURE Q1(a)

FINAL EXAMINATION

SEMESTER/SESSION: SEM I/2013/2014 PROGRAMME : 3 BDD
COURSE NAME : ENGINEERING MATHEMATICS IV COURSE CODE: BWM30603/BSM3913

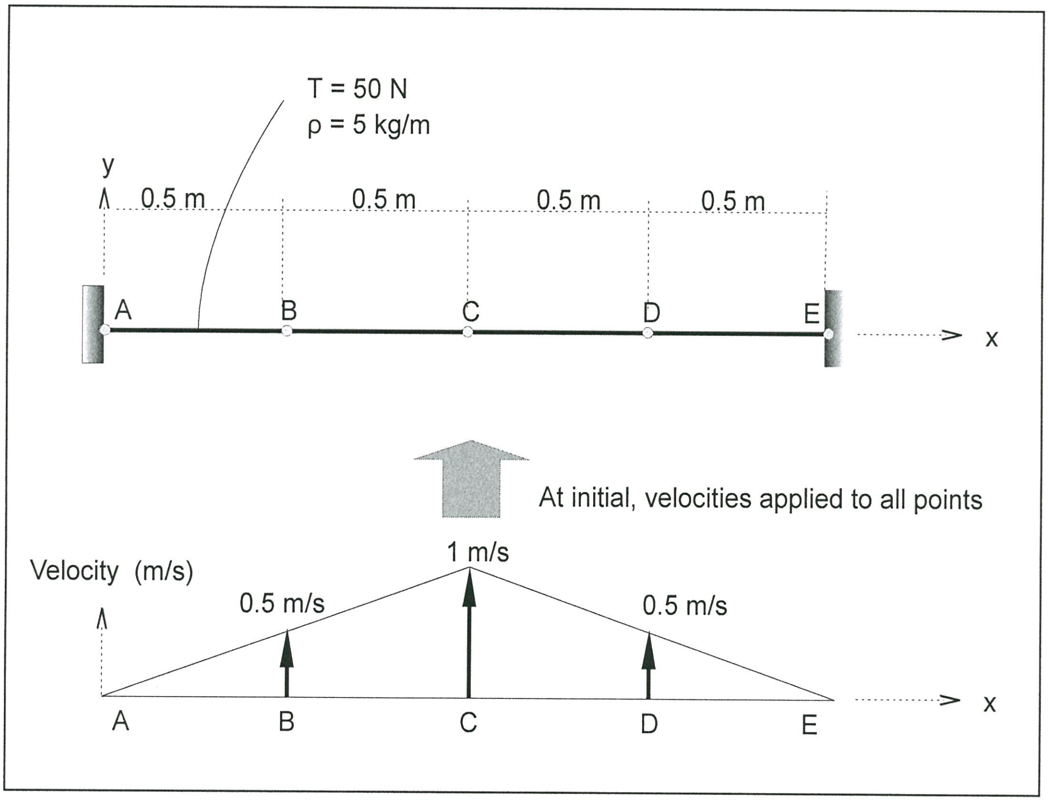

FIGURE Q1 (b)

FINAL EXAMINATION

SEMESTER/SESSION: SEM I/2013/2014 PROGRAMME : 3 BDD
COURSE NAME : ENGINEERING MATHEMATICS IV COURSE CODE: BWM30603/BSM3913

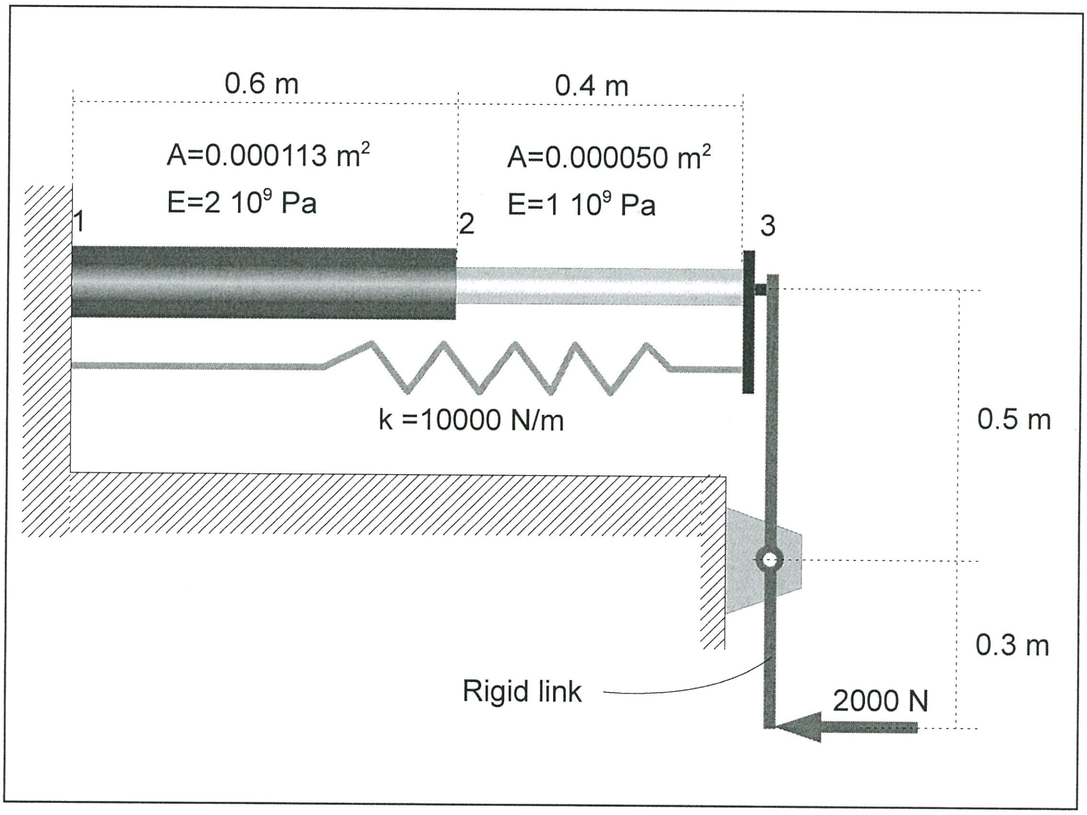

FIGURE Q2

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SEMESTER/SESSION: SEM I/2013/2014 PROGRAMME : 3 BDD
COURSE NAME : ENGINEERING MATHEMATICS IV COURSE CODE: BWM30603/BSM3913

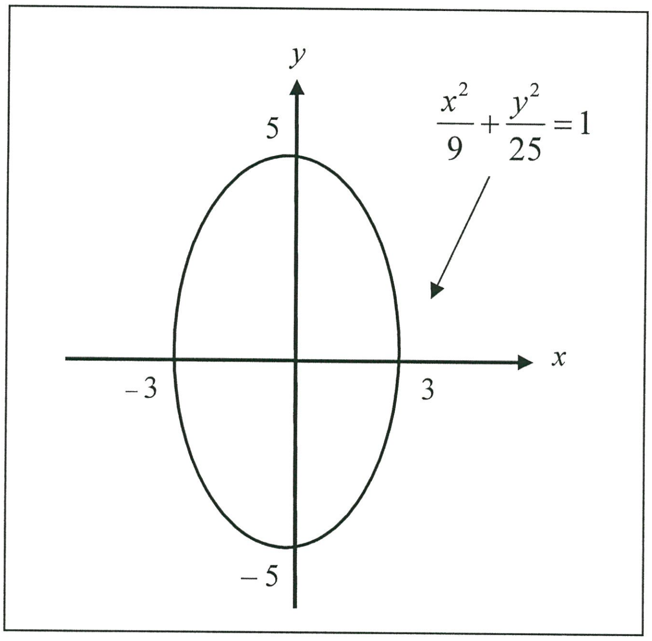

FIGURE Q4 (c)

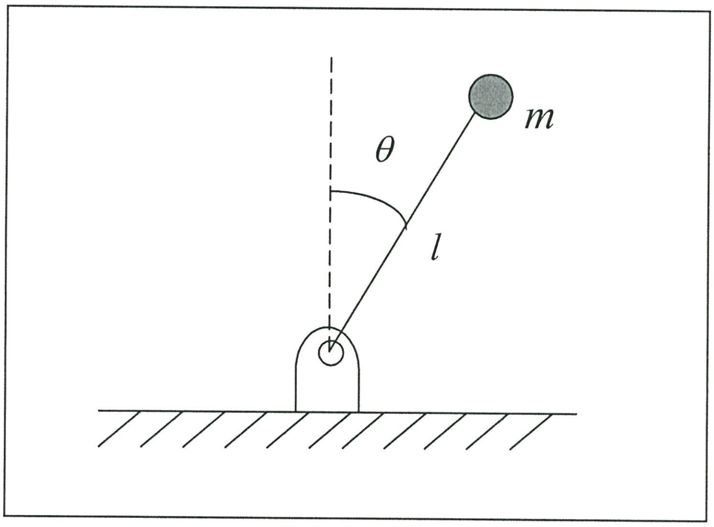

FIGURE Q5 (c)

FINAL EXAMINATION

SEMESTER/SESSION: SEM I/2013/2014 PROGRAMME : 3 BDD
COURSE NAME : ENGINEERING MATHEMATICS IV COURSE CODE: BWM30603/BSM3913

FORMULAS

Nonlinear equations

Bisection method: $c_i = \dfrac{a_i + b_i}{2}, \quad i = 0,1,2,...,n$

System of linear equations

Gauss-Seidel iteration method: $x_i^{(k+1)} = \dfrac{b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)}}{a_{ii}}, i = 1,2,\ldots,n$

Interpolation

Lagrange polynomial:

$$P_n(x) = \sum_{i=0}^{n} L_i(x) f(x_i), i = 0,1,2,\ldots,n \text{ where } L_i(x) = \prod_{\substack{j=0 \\ j\neq i}}^{n} \frac{(x - x_j)}{(x_i - x_j)}$$

Numerical differentiation and integration

Differentiation:

First derivatives:

3-point central difference: $f'(x) \approx \dfrac{f(x+h) - f(x-h)}{2h}$

Second derivatives:

3-point central difference: $f''(x) \approx \dfrac{f(x+h) - 2f(x) + f(x-h)}{h^2}$

Integration:

Simpson's $\frac{1}{3}$ rule: $\int_a^b f(x)\, dx \approx \dfrac{h}{3}\left[f_0 + f_n + 4\sum_{\substack{i=1 \\ i\,\text{odd}}}^{n-1} f_i + 2\sum_{\substack{i=2 \\ i\,\text{even}}}^{n-2} f_i \right]$

Simpson's $\frac{3}{8}$ rule:

$$\int_a^b f(x)\, dx \approx \frac{3}{8} h\left[f_0 + f_n + 3(f_1 + f_2 + f_4 + f_5 + \ldots + f_{n-2} + f_{n-1}) + 2(f_3 + f_6 + \ldots + f_{n-3}) \right]$$

FINAL EXAMINATION

SEMESTER/SESSION: SEM I/2013/2014 PROGRAMME : 3 BDD
COURSE NAME : ENGINEERING MATHEMATICS IV COURSE CODE: BWM30603/BSM3913

Eigenvalue

Power Method: $v^{(k+1)} = \frac{1}{m_{k+1}} A v^{(k)}, \quad k = 0, 1, 2, \ldots$

Ordinary differential equations

Initial value problems:

Euler's method: $y_{i+1} = y_i + hf(x_i, y_i)$

Modified Euler's method: $y_{i+1} = y_i + \frac{1}{2}k_1 + \frac{1}{2}k_2$

where $k_1 = hf(x_i, y_i)$, $k_2 = hf(x_i + h, y_i + k_1)$

Boundary value problems:

Finite-difference method: $y_i' \approx \frac{y_{i+1} - y_{i-1}}{2h}$, $y_i'' \approx \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$

Partial differential equation

Heat Equation: Explicit finite difference method

$$\left(\frac{\partial u}{\partial t}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \Leftrightarrow \frac{u_{i,j+1} - u_{i,j}}{k} = c^2 \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

Wave equation- Finite difference method:

$$\left(\frac{\partial^2 u}{\partial t^2}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \Leftrightarrow \frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{k^2} = c^2 \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

$$\frac{\partial u(x,0)}{\partial t} = \frac{u_{i,j+1} - u_{i,j-1}}{2k} = g(x_i)$$