UTHM
Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2013/2014

COURSE NAME	:	CALCULUS
COURSE CODE	:	BWC 10303
PROGRAMME	:	1 BWC
EXAMINATION DATE	:	DECEMBER 2013/JANUARY 2014
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS.

THIS QUESTION PAPER CONSISTS OF FIVE **(5)** PAGES

Q1 (a) Given

$$f(x) = \begin{cases} x - 2, & x < 0 \\ x^2, & x \geq 0 \end{cases}.$$

Find

(i) $\lim_{x \to 0} f(x)$.

(ii) $f(0)$.

(iii) Is the function continuous at $x = 0$? Justify your answer.

(4 marks)

(b) Differentiate the following functions with respect to x.

(i) $y = \dfrac{\cos x}{1 + \sin x}$.

(2 marks)

(ii) $y = x^{\ln x}$.

(5 marks)

(c) If $(\cos x)y^2 - 2x^2 = e^{xy}$, find $\dfrac{dy}{dx}$ in terms of x and y.

(9 marks)

Q2 (a) Given $f(x) = 9x^{\frac{2}{3}}$.

(i) Write the expression of $f'(x)$.
(ii) Find the value of x when $f(x) = 36$.
(iii) Hence, find the approximate value of x when $f(x)$ increases from 36.0 to 36.4.

(12 marks)

(b) Let $f(x) = (x^2 - 1)^3$.

(i) Find all critical points of $f(x)$.
(ii) Hence, determine whether the critical points is minimum, maximum or inflection point.

(8 marks)

Q3 (a) Evaluate these integrals.

(i) $\int \dfrac{dx}{x^{-6}}$.

(ii) $\int \dfrac{e^{5x}}{e^{3x}} dx$.

(iii) $\int_0^\pi \dfrac{\sin 2x}{2\cos x} dx$.

(10 marks)

(b) Given $y = x\sqrt{x+1}$.

(i) Show that $\dfrac{dy}{dx} = \dfrac{3x+2}{2\sqrt{x+1}}$.

(ii) Hence, evaluate $\int_3^8 \dfrac{3x+2}{\sqrt{x+1}} dx$.

(10 marks)

Q4 (a) By using proper substitution, evaluate $\int \dfrac{x^2}{\sqrt[4]{x^3+2}} dx$.

(5 marks)

(b) Find $\int x^2 e^{-x} dx$.

(5 marks)

(c) Show that $\int \dfrac{3x+5}{(x+1)(x-1)^2} dx = \dfrac{1}{2}\ln\left|\dfrac{x+1}{x-1}\right| + \dfrac{4}{x-1} + c$.

(10 marks)

Q5 (a) Find $f'(x)$ for $f(x) = \sin^{-1}(\ln x)$.

(5 marks)

(b) Find area of the surface that is generated by revolving the curve $y = \sqrt[3]{3x}$ between $y = -1$ and $y = 0$ about the y-axis.

(10 marks)

(c) Find the curvature if $x = \cos t$ and $y = \ln 2t$ at $t = \pi$.

(5 marks)

- END OF QUESTION -

FINAL EXAMINATION

SEMESTER / SESSION : SEM I / 2013/2014   PROGRAMME :1 BWC

COURSE : CALCULUS   COURSE CODE : BWC 10303

Formulae

Indefinite Integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \csc^2 x \, dx = -\cot x + C$$

$$\int \sec x \tan x \, dx = \sec x + C$$

$$\int \csc x \cot x \, dx = -\csc x + C$$

$$\int e^x dx = e^x + C$$

$$\int \cosh x \, dx = \sinh x + C$$

$$\int \sinh x \, dx = \cosh x + C$$

$$\int \operatorname{sech}^2 x \, dx = \tanh x + C$$

$$\int \operatorname{csch}^2 x \, dx = -\coth x + C$$

$$\int \operatorname{sech} x \tanh x \, dx = -\operatorname{sech} x + C$$

$$\int \operatorname{csch} x \coth x \, dx = -\operatorname{csch} x + C$$

Integration of Inverse Functions

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C, \ |x| < 1$$

$$\int \frac{-1}{\sqrt{1-x^2}} dx = \cos^{-1} x + C, \ |x| < 1$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$$

$$\int \frac{-1}{1+x^2} dx = \cot^{-1} x + C$$

$$\int \frac{1}{|x|\sqrt{x^2-1}} dx = \sec^{-1} x + C, \ |x| > 1$$

$$\int \frac{-1}{|x|\sqrt{x^2-1}} dx = \csc^{-1} x + C, \ |x| > 1$$

$$\int \frac{1}{\sqrt{x^2+1}} dx = \sinh^{-1} x + C$$

$$\int \frac{1}{\sqrt{x^2-1}} dx = \cosh^{-1} x + C, \ |x| > 1$$

$$\int \frac{-1}{|x|\sqrt{1-x^2}} dx = \operatorname{sech}^{-1}|x| + C, \ 0 < x < 1$$

$$\int \frac{-1}{|x|\sqrt{1+x^2}} dx = \operatorname{csch}^{-1}|x| + C, \ x \neq 0$$

$$\int \frac{1}{1-x^2} dx = \begin{cases} \tanh^{-1} x + C, & |x| < 1 \\ \coth^{-1} x + C, & |x| > 1 \end{cases}$$

TRIGONOMETRIC SUBSTITUTION

Expression	Trigonometry	Hyperbolic
$\sqrt{x^2 + k^2}$	$x = k \tan \theta$	$x = k \sinh \theta$
$\sqrt{x^2 - k^2}$	$x = k \sec \theta$	$x = k \cosh \theta$
$\sqrt{k^2 - x^2}$	$x = k \sin \theta$	$x = k \tanh \theta$

FINAL EXAMINATION

SEMESTER / SESSION : SEM I / 2013/2014   PROGRAMME :1 BWC

COURSE : CALCULUS   COURSE CODE : BWC 10303

Formulae

TRIGONOMETRIC SUBSTITUTION

$t = \tan \frac{1}{2} x$		$t = \tan x$	
$\sin x = \dfrac{2t}{1+t^2}$	$\cos x = \dfrac{1-t^2}{1+t^2}$	$\sin 2x = \dfrac{2t}{1+t^2}$	$\cos 2x = \dfrac{1-t^2}{1+t^2}$
$\tan x = \dfrac{2t}{1-t^2}$	$dx = \dfrac{2dt}{1+t^2}$	$\tan 2x = \dfrac{2t}{1-t^2}$	$dx = \dfrac{dt}{1+t^2}$

IDENTITIES OF TRIGONOMETRY AND HYPERBOLIC

Trigonometric Functions	*Hyperbolic Functions*
$\cos^2 x + \sin^2 x = 1$	$\sinh x = \dfrac{e^x - e^{-x}}{2}$
$\sin 2x = 2 \sin x \cos x$	
$\cos 2x = \cos^2 x - \sin^2 x$	$\cosh x = \dfrac{e^x + e^{-x}}{2}$
$\quad = 2 \cos^2 x - 1$	$\cosh^2 x - \sinh^2 x = 1$
$\quad = 1 - 2 \sin^2 x$	$\sinh 2x = 2 \sinh x \cosh x$
$1 + \tan^2 x = \sec^2 x$	$\cosh 2x = \cosh^2 x + \sinh^2 x$
$1 + \cot^2 x = \csc^2 x$	$\quad = 2 \cosh^2 x - 1$
$\tan 2x = \dfrac{2 \tan x}{1 - \tan^2 x}$	$\quad = 1 + 2 \sinh^2 x$
$\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$	$1 - \tanh^2 x = \operatorname{sech}^2 x$
$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$	$\coth^2 x - 1 = \operatorname{csch}^2 x$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$	$\tanh 2x = \dfrac{2 \tanh x}{1 + \tanh^2 x}$
$2 \sin ax \cos bx = \sin(a+b)x + \sin(a-b)x$	$\tanh(x \pm y) = \dfrac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$
$2 \sin ax \sin bx = \cos(a-b)x - \cos(a+b)x$	$\sinh(x \pm y) = \sinh x \cosh y \pm \sinh y \cosh x$
$2 \cos ax \cos bx = \cos(a-b)x + \cos(a+b)x$	$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$

CURVATURE, ARC LENGTH AND SURFACE AREA OF REVOLUTION

$$\kappa = \frac{\left| \dfrac{d^2 y}{dx^2} \right|}{\left[1 + \left(\dfrac{dy}{dx} \right)^2 \right]^{3/2}}$$

$$\kappa = \frac{| \dot{x}\ddot{y} - \dot{y}\ddot{x} |}{\left[\dot{x}^2 + \dot{y}^2 \right]^{3/2}}$$

$$L = \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx} \right)^2} \, dx$$

$$L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2} \, dt$$

$$L = \int_{y_1}^{y_2} \sqrt{1 + \left(\frac{dx}{dy} \right)^2} \, dy$$

$$S = 2\pi \int_{x_1}^{x_2} f(x) \sqrt{1 + \left(\frac{d}{dx} [f(x)] \right)^2} \, dx$$

$$S = 2\pi \int_{y_1}^{y_2} g(y) \sqrt{1 + \left(\frac{d}{dy} [g(y)] \right)^2} \, dy$$