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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2013/2014

COURSE NAME	:	MATHEMATICS FOR ENGINEERING TECHNOLOGY III
COURSE CODE	:	BWM 22403
PROGRAMME	:	2 BND/BNH/BNK/BNL
EXAMINATION DATE	:	DECEMBER 2013/JANUARY 2014
DURATION	:	3 HOURS
INSTRUCTION	:	A) ANSWER **ALL FIVE (5)** QUESTIONS. B) ALL CALCULATIONS MUST BE IN **3 DECIMAL** PLACES.

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

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Q1 (a) Given that the dominant eigenvalue λ_{largest} is 13.262. Find the smallest (in absolute) eigenvalue for matrix A below by using shifted power method with $v^{(0)} = \begin{pmatrix}1 & 1 & 1\end{pmatrix}^T$.

$$A = \begin{pmatrix} 8 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 13 \end{pmatrix}$$

(9 marks)

(b) Given the heat equation

$$\frac{\partial u}{\partial t} = 0.5\frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 1, \quad t > 0,$$

with boundary conditions $u(0,t) = 20e^{-t}$ and $u(1,t) = 60e^{-2t}$ for $t > 0$ and initial condition $u(x,0) = 20 + 40x$ for $0 \le x \le 1$. By using implicit Crank-Nicolson method, solve the heat equation at first level only for $t \le 0.1$ by taking $\Delta x = h = 0.25$, and $\Delta t = k = 0.1$ by using calculator.

(11 marks)

Q2 (a) Given $f(x) = \sqrt{\cot x}$. Find the approximate value(s) of $f'(0.05)$ with $h = 0.01$ by using

(i) 2 – point backward difference formula,

(1 mark)

(ii) 3 – point central difference formula,

(1 mark)

(iii) 3 – point forward difference formula,

(1 mark)

(iv) 5 – point difference formula.

(1 mark)

Then, find the relative error for each answer if the exact answer is –44.777.

(2 marks)

(b) Suppose that the age in days of a type of single-celled organism can be expressed as $f(x) = (\ln 2)e^{-xk}$ where $k = \frac{1}{2}\ln 2$ and the domain is $0 \le x \le 2$. Given that mean $= \mu = \int_0^2 f(x)\,dx$, find the mean age of the cells by using

(i) 1/3 Simpson method with $h = 0.2$.

(6 mark)

(ii) 2-point Gauss quadrature.

(8 marks)

Q3 (a) Given the following function

$$\frac{\pi h^2 (3r - h)}{3V},$$

where the volume V of liquid in a spherical tank of radius r is related to the depth h of the liquid. From past experiences, the value of h is in the interval [3.2m, 3.5m]. Find h using Newton-Raphson method when $r = 1$ m and $V = 0.5\text{m}^3$.

(9 marks)

(b) Given the system of linear equations

$$\begin{aligned} 2x_1 + 5x_2 + 2x_3 &= 8 \\ 5x_1 + 2x_2 &= -2 \\ 2x_2 + 5x_3 &= 3. \end{aligned}$$

Solve the system by using Gauss-Seidel iteration method. Take initial guess as $x^{(0)} = (-1.220 \quad 2.176 \quad -0.270)^T$ and iterate until $\max\left\{ |x_i^{(k+1)} - x_i^{(k)}| \right\} < \varepsilon = 0.005$.

(11 marks)

Q4 (a) Solve the system of linear equations below by using Thomas algorithm method.

$$\begin{pmatrix} 5 & 10 & -4 & 0 \\ 10 & 5 & 0 & 0 \\ 0 & -4 & 8 & -1 \\ 0 & 0 & -1 & 5 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 25 \\ 6 \\ -11 \\ -11 \end{pmatrix}$$

(13 marks)

(b) A car travelling along a rural highway has been clocked at a number of points. The data from the observations are given in the Table **Q4(b)**, where the time is in second s and the distance is in metre m.

Table Q4(b): Observation of a car travelling along a rural highway

Time t	0	3	5	8	13
Distance d	0	70	116	190	303

Use Newton divided difference method to predict the position of the car when $t = 10\,s$.

(7 marks)

Q5 (a) Find the partial derivatives f_x, f_y and f_{xx} of the function

$$f(x, y) = e^{xy}\sin(4y^2).$$

(3 marks)

(b) Find the local extreme and saddle point(s) of

$$f(x, y) = x^3 + y^3 - 3xy.$$

(7 marks)

(c) Evaluate

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{\sqrt{x^2+y^2}}^{1} dz\,dy\,dx$$

by changing it to spherical coordinate.

(10 marks)

- END OF QUESTION -

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Formulae

Nonlinear equations

Newton-Raphson method : $x_{i+1} = x_i - \dfrac{f(x_i)}{f'(x_i)}$, $i = 0,1,2,\ldots$

System of linear equations

Thomas Algorithm

i	1	2	…	n
d_i				
e_i				
c_i				
b_i				
$\alpha_1 = d_1$ $\alpha_i = d_i - c_i\beta_{i-1}$				
$\beta_i = \dfrac{e_i}{\alpha_i}$				
$y_1 = \dfrac{b_1}{\alpha_1}$ $y_i = \dfrac{b_i - c_i y_{i-1}}{\alpha_i}$				
$x_n = y_n$ $x_i = y_i - \beta_i x_{i+1}$				

Gauss-Seidel iteration method: $x_i^{(k+1)} = \dfrac{b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)}}{a_{ii}}, i = 1,2,\ldots,n$

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Interpolation

Newton divided difference:

$$P_n(x) = f_0^{[0]} + f_0^{[1]}(x - x_0) + f_0^{[2]}(x - x_0)(x - x_1) + \ldots + f_0^{[n]}(x - x_0)(x - x_1)\ldots(x - x_{n-1})$$

Numerical differentiation and integration

Differentiation:

First derivatives:

2-point forward difference: $f'(x) \approx \dfrac{f(x+h) - f(x)}{h}$

2-point backward difference: $f'(x) \approx \dfrac{f(x) - f(x-h)}{h}$

3-point forward difference: $f'(x) \approx \dfrac{-f(x+2h) + 4f(x+h) - 3f(x)}{2h}$

3-point backward difference: $f'(x) \approx \dfrac{3f(x) - 4f(x-h) + f(x-2h)}{2h}$

3-point central difference: $f'(x) \approx \dfrac{f(x+h) - f(x-h)}{2h}$

5-point difference: $f'(x) \approx \dfrac{-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)}{12h}$

Integration:

$\dfrac{1}{3}$ Simpson's rule: $\displaystyle\int_a^b f(x)dx \approx \frac{h}{3}\left[f_0 + f_n + 4\sum_{\substack{i=1 \\ i \text{ odd}}}^{n-1} f_i + 2\sum_{\substack{i=2 \\ i \text{ even}}}^{n-2} f_i \right]$

Gauss quadrature: For $\displaystyle\int_a^b f(x)dx$, $\quad x = \dfrac{(b-a)t + (b+a)}{2}$

2-points: $\displaystyle\int_{-1}^{1} f(x)dx \approx g\left(-\frac{1}{\sqrt{3}}\right) + g\left(\frac{1}{\sqrt{3}}\right)$

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Eigen value

Power Method : $\mathbf{v}^{(k+1)} = \dfrac{1}{m_{k+1}} A\mathbf{v}^{(k)}, \quad k = 0,1,2,.....$

Shifted Power Method: $\mathbf{A}_{\text{shifted}} = \mathbf{A} - s\mathbf{I}, \ \lambda_{\text{smallest}} = \lambda_{\text{Shifted}} + s$

Partial differential equations

Heat equation - Implicit Crank-Nicolson:

$$\left(\frac{\partial u}{\partial t}\right)_{i,j+\frac{1}{2}} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j+\frac{1}{2}}$$

$$\frac{u_{i,j+1} - u_{i,j}}{k} = \frac{c^2}{2}\left(\frac{u_{i-1,j+1} - 2u_{i,j+1} + u_{i+1,j+1}}{h^2} + \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}\right)$$

Local extreme value

$$G(x,y) = f_{xx}(x,y) \times f_{yy}(x,y) - \left[f_{xy}(x,y)\right]^2$$

$G(a,b) > 0$ and $f_{xx}(a,b) < 0$ $\Rightarrow$ local maximum value

$G(a,b) > 0$ and $f_{xx}(a,b) > 0$ $\Rightarrow$ local minimum value

$G(a,b) < 0$ $\Rightarrow$ saddle point

$G(a,b) = 0$ $\Rightarrow$ test is inconclusive

Cartesian coordinate to spherical coordinate

$$x = \rho\cos\theta\sin\phi, \ y = \rho\sin\theta\sin\phi, \ z = \rho\cos\phi, \ x^2 + y^2 + z^2 = \rho^2, \ 0 \le \theta \le 2\pi, \ 0 \le \phi \le \pi$$

$$\iiint_G f(x,y,z)\, dV = \iiint_G f(\rho,\phi,\theta)\, \rho^2 \sin\phi \, d\rho\, d\phi\, d\theta$$