UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2013/2014

COURSE NAME	:	MATHEMATICS IV
COURSE CODE	:	BWM21403
PROGRAMME	:	PPG
EXAMINATION DATE	:	DECEMBER 2013/JANUARY 2014
DURATION	:	3 HOURS
INSTRUCTION	:	A)ANSWER **FOUR (4)** QUESTIONS ONLY B)DO YOUR CALCULATION IN **3** DECIMAL PLACES

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

Q1 (a) Solve the differential equation $x^3dy - 2ydx = 0$ with the initial condition $y(1) = \frac{1}{e}$.

(12 marks)

(b) Find the general solution for linear differential equation $\frac{dy}{dx} - \frac{4}{x}y = x^4$.

(13 marks)

Q2 Find the particular solution for initial value problem $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} = 1 - x$ with initial conditions $y(0) = 1,\ y'(0) = 1$

(25 marks)

Q3 (a) Find the root of $x + \ln x = 0$ by using bisection method. Iterate until $|f(c_i)| < \varepsilon = 0.005$ over the interval $[0.2, 1]$.

(15 marks)

(b) Given the graph of $f(x) = 2x^3 - 5x^2 - 7x + 6$ as in figure **Q3(b).**

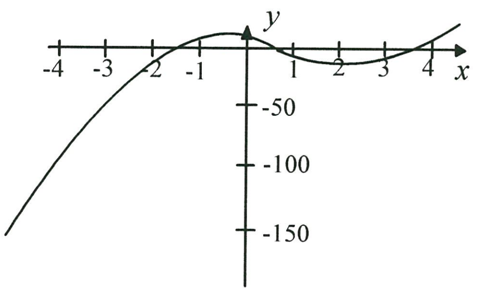

FIGURE Q3(b)

Find the most positive root of $f(x)$ by using Newton-Raphson method. Use $x_0 = 3$. Iterate until $|f(c_i)| < \varepsilon = 0.005$.

(10 marks)

Q4 Given the system of linear equations as below $Ax = b$

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & 3 & 5 \\ -2 & 4 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix}$$

(a) Write $A = LU$ as LU defined by Doolittle method.

(13 marks)

(b) From $LY = b$, solve for Y by forward substitution.

(6 marks)

(c) From $Ux = Y$, solve for x by backward substitution.

(6 marks)

Q5 With $n = 8$ subintervals, find the approximate value for

$$\int_0^{\pi} \sin x \, dx$$

using

(a) trapezoidal rule

(13 marks)

(b) $\frac{1}{3}$ Simpson's rule.

(12 marks)

Q6 (a) Find Lagrange interpolating polynomial for data as shown in the table **Q6(a)**.

TABLE Q6(a)

x	0	2	4	6
$f(x)$	1	-1	-1	1

Hence, find the value $f(5)$.

(12 marks)

(b) Find the Newton's interpolatory divided-difference polynomial for data as shown in the table **Q6(b)**.

TABLE Q6(b)

x	2.0	2.1	2.4	2.6
$f(x)$	0.510	0.521	0.510	0.381

Hence, find the value $f(2.5)$.

(13 marks)

Q7 Solve the following first-order initial value problem at $x = 0(0.2)1$ by Euler's method.

$$2\frac{dy}{dx} + 3y = e^{2x}, \text{ with initial condition } y(0) = 1.$$

Given the exact solution is $y(x) = \frac{1}{7}e^{2x} + \frac{6}{7}e^{-\frac{3x}{2}}$. Find the errors.

(25 marks)

- END OF QUESTION -

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FORMULAS

Second-order Differential Equation

Characteristic equation: $am^2 + bm + c = 0$.		
Case	The roots of characteristic equation	General solution
1.	Real and different roots: m_1 and m_2	$y = Ae^{m_1 x} + Be^{m_2 x}$
2.	Real and equal roots: $m = m_1 = m_2$	$y = (A + Bx)e^{mx}$
3.	Complex roots: $m_1 = \alpha + \beta i$, $m_2 = \alpha - \beta i$	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Nonlinear Equations

Bisection : $c_i = \dfrac{a_i + b_i}{2}$, $i = 0,1,2,...$

Newton-Raphson formula : $x_{i+1} = x_i - \dfrac{f(x_i)}{f'(x_i)}$, $i = 0, 1, 2, ...$

System of Linear Equations

Crout Factorization: $A = LU$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{pmatrix} \begin{pmatrix} 1 & g & h \\ 0 & 1 & i \\ 0 & 0 & 1 \end{pmatrix}$$

Doolittle Factorization: $A = LU$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{pmatrix} \begin{pmatrix} d & e & f \\ 0 & g & h \\ 0 & 0 & i \end{pmatrix}$$

Cholesky Factorization: $A = LU$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{pmatrix} \begin{pmatrix} a & b & d \\ 0 & c & e \\ 0 & 0 & f \end{pmatrix}$$

Gauss-Seidel iteration : $x_i^{(k+1)} = \dfrac{b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)}}{a_{ii}}$, $\forall i = 1, 2, 3,.., n$.

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Interpolation

Lagrange interpolation : $P_n(x) = \sum_{i=0}^{n} L_i(x) f_i$ for $k = 0,1,2,3,\ldots,n$ with $L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{(x - x_j)}{(x_i - x_j)}$

Newton's interpolatory divided-difference formula:

$$P_n(x) = f_0^{[0]} + f_0^{[1]}(x - x_0) + f_0^{[2]}(x - x_0)(x - x_1) + \cdots + f_0^{[n]}(x - x_0)(x - x_1)\cdots(x - x_{n-1})$$

Numerical Integration

Trapeziod rule : $\int_a^b f(x)\, dx \approx \frac{h}{2}\left(f_0 + f_n + 2\sum_{i=1}^{n-1} f_i \right)$

Simpson $\frac{1}{3}$ Rule: $\int_a^b f(x)\, dx \approx \frac{h}{3}\left[f_0 + f_n + 4\sum_{\substack{i=1 \\ i\,\text{odd}}}^{n-1} f_i + 2\sum_{\substack{i=2 \\ i\,\text{even}}}^{n-2} f_i \right]$

Ordinary Differential Equation
Initial Value Problem:

Taylor Series Method: $y(x_{i+1}) = y(x_i) + hy'(x_i) + \frac{h^2}{2!} y''(x_i) + \cdots + \frac{h^n}{n!} y^{(n)}(x_i)$

Classical Fourth-order Runge-Kutta Method: $y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

where $k_1 = hf(x_i, y_i)$ $\quad$ $k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2})$

$k_3 = hf(x_i + \frac{h}{2}, y_i + \frac{k_2}{2})$ $\quad$ $k_4 = hf(x_i + h, y_i + k_3)$