Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER 1
SESSION 2013/2014

COURSE NAME	:	ORDINARY DIFFERENTIAL EQUATIONS
COURSE CODE	:	BWA 20303
PROGRAMME	:	2 BWA
EXAMINATION DATE	:	DECEMBER 2013/JANUARY 2014
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS IN **PART A** AND **THREE (3)** QUESTIONS IN **PART B**

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

PART A

Q1 Given

$$y''-2xy=0.$$

(a) By assuming $y=\sum_{0}^{\infty} c_m x^m$, show that the differential equation above can be expressed as

$$2c_2+\sum_{n=1}^{\infty}[(n+2)(n+1)c_{n+2}-2c_{n-1}]x^n.$$

(6 marks)

(b) Show that the recurrence relation is given by

$$c_2=0, \quad \text{and} \quad c_{n+2}=-\frac{2c_{n-1}}{(n+2)(n+1)}, \quad n=1, 2, 3, \ldots$$

(2 marks)

(c) Deduce the coefficient of series c_n, for $n=1, 2, 3, \ldots, 7$ in terms of c_0 and c_1.

(6 marks)

(d) Verify that the general solution of the differential equation is

$$y(x)=c_0\left[1+\frac{2}{3\cdot 2}x^3+\frac{2^2}{6\cdot 5\cdot 3\cdot 2}x^6+\ldots\right]+c_1\left[x+\frac{2}{4\cdot 3}x^4+\frac{2^2}{7\cdot 6\cdot 4\cdot 3}x^7+\ldots\right].$$

(6 marks)

Q2 Given the system of first order differential equation

$$x'_1=x_1+2x_2$$
$$x'_2=2x_1+4x_2$$

(a) Write the equation in matrix form of $X'=AX$, where A is the coefficient matrix.

(2 marks)

(b) Show that the eigenvalues are $\lambda=0$ and $\lambda=5$.

(5 marks)

(c) Find the corresponding eigenvectors for the eigenvalues found in Q2(b).

(6 marks)

(d) Determine whether the corresponding eigenvectors are linearly independent or not.

(4 marks)

(e) Verify that the general solution is given by

$$x(t) = C_1 \begin{pmatrix} 1 \\ 1 \\ -\frac{1}{2} \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{5t}.$$

(3 marks)

PART B

Q3 (a) (i) Given $y = e^x \sin x$. Show that $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + 2y = 0$.

(4 marks)

(ii) Solve $(x + \sin y)dx + (x\cos y - 2y)dy = 0$.

(6 marks)

(b) Solve the following differential equation by using the method of integrating factor.

$$x\frac{dy}{dx} - y = x$$

(4 marks)

(c) By using a substitution of $y = vx$, and $\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$, find the solution of

$$y' = \frac{x^2 + y^2}{xy}, \quad y(1) = 1.$$

(6 marks)

Q4 (a) Find the particular solution for the second order differential equation

$$y'' + 2y' - 8y = 0, \qquad y(0) = 5, \qquad y'(0) = -12.$$

(6 marks)

(b) Find the general solution for the second order differential equation

$$y'' + 2y' + 5y = x^2 - 1$$

by using the undetermined coefficient method.

(8 marks)

(c) Given a non-homogeneous second order differential equation

$$y'' - 6y' + 9y = xe^{3x}.$$

Find the general solution for the equation by using variation of parameters method.

(6 marks)

Q5 (a) Find the following transforms.

(i) $L\{(1-e^{-t})^2\}$,

(ii) $L^{-1}\left\{\dfrac{24}{(s-3)^5}\right\}$.

(5 marks)

(b) Consider the function

$$f(t)=\begin{cases} e^t, & 0\le t<2, \\ t-2, & t\ge 2. \end{cases}$$

(i) Write the function $f(x)$ in the form of unit step function.

(ii) Find the Laplace transform of $f(t)$.

(9 marks)

(c) By using Laplace Transform, solve $y''-6y'+9y=t^2e^{3t}$, subject to $y(0)=2$ and $y'(0)=6$.

(6 marks)

Q6 (a) Show that $L^{-1}\left\{\dfrac{4-s}{s^3(s+1)}\right\}=5-5t+2t^2-5e^{-t}$.

(8 marks)

(b) By using Laplace transform, solve the system of linear differential equations

$$\begin{aligned} 2x'+y'-y&=t \\ x'+y'&=t^2 \end{aligned}$$

subject to $x(0)=1,\quad y(0)=0$.

(12 marks)

–END OF QUESTION –

FINAL EXAMINATION

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FORMULA

Second-order Differential Equation

The roots of characteristic equation and the general solution for differential equation $ay'' + by' + cy = 0$ or $a\ddot{y} + b\dot{y} + cy = 0$ or $a\dfrac{d^2y}{dx^2} + b\dfrac{dy}{dx} + cy = 0$.

Characteristic equation: $am^2 + bm + c = 0$.		
Case	The roots of characteristic equation	General solution
1.	Real and different roots: m_1 and m_2	$y = Ae^{m_1x} + Be^{m_2x}$
2.	Real and equal roots: $m = m_1 = m_2$	$y = (A + Bx)e^{mx}$
3.	Complex roots: $m_1 = \alpha + \beta i$, $m_2 = \alpha - \beta i$	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

The method of undetermined coefficients

For non-homogeneous second order differential equation $ay'' + by' + cy = f(x)$, the particular solution is given by $y_p(x)$:

$f(x)$	$y_p(x)$
$P_n(x) = A_nx^n + A_{n-1}x^{n-1} + \cdots + A_1x + A_0$	$x^r(B_nx^n + B_{n-1}x^{n-1} + \cdots + B_1x + B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(P\cos\beta x + Q\sin\beta x)$
$P_n(x)e^{\alpha x}$	$x^r(B_nx^n + B_{n-1}x^{n-1} + \cdots + B_1x + B_0)e^{\alpha x}$
$P_n(x)\begin{cases}\cos\beta x\\ \sin\beta x\end{cases}$	$x^r(B_nx^n + B_{n-1}x^{n-1} + \cdots + B_1x + B_0)\cos\beta x +$ $x^r(C_nx^n + C_{n-1}x^{n-1} + \cdots + C_1x + C_0)\sin\beta x$
$Ce^{\alpha x}\begin{cases}\cos\beta x\\ \sin\beta x\end{cases}$	$x^re^{\alpha x}(P\cos\beta x + Q\sin\beta x)$
$P_n(x)e^{\alpha x}\begin{cases}\cos\beta x\\ \sin\beta x\end{cases}$	$x^r(B_nx^n + B_{n-1}x^{n-1} + \cdots + B_1x + B_0)e^{\alpha x}\cos\beta x +$ $x^r(C_nx^n + C_{n-1}x^{n-1} + \cdots + C_1x + C_0)e^{\alpha x}\sin\beta x$

Note : r is the least non-negative integer (r = 0, 1, or 2) which determine such that there is no terms in particular integral $y_p(x)$ corresponds to the complementary function $y_c(x)$.

The method of variation of parameters

If the solution of the homogeneous equation $ay'' + by' + cy = 0$ is $y_c = Ay_1 + By_2$, then the particular solution for $ay'' + by' + cy = f(x)$ is $y = y_c + y_p$, and $y_p = uy_1 + vy_2$,

where $u = -\int \dfrac{y_2 f(x)}{aW}dx$, $\quad v = \int \dfrac{y_1 f(x)}{aW}dx$ and $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1y_2' - y_2y_1'$.

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Laplace Transform

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
e^{at}	$\frac{1}{s-a}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
$\sin at$	$\frac{a}{s^2+a^2}$	$\delta(t-a)$	e^{-as}
$\cos at$	$\frac{s}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$\int_0^t f(u)g(t-u)\,du$	$F(s)\cdot G(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$y(t)$	$Y(s)$
$t^n,\ n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	$y'(t)$	$sY(s)-y(0)$
$e^{at}f(t)$	$F(s-a)$	$y''(t)$	$s^2Y(s)-sy(0)-y'(0)$
$t^n f(t),\ n=1,2,3,\ldots$	$(-1)^n \frac{d^n}{ds^n}F(s)$		

Representation of Functions in Power Series

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots..$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots..$$