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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2013/2014

COURSE NAME	:	STATISTICS AND PROBABILITY II
COURSE CODE	:	BWB 10303
PROGRAMME	:	2 BWA
EXAMINATION DATE	:	DECEMBER 2013/ JANUARY 2014
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **FIVE (5)** PAGES

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Q1 (a) Explain the scenario of Bernoulli trials and Binomial distribution.

(4 marks)

(b) Mathematically, the moment generating function of the geometric distribution is

$$M(t) = \frac{\theta e^t}{1 - e^t(1-\theta)}.$$

By using that moment generating function, derive the variance of geometric distribution. (Hint: $q = 1-\theta$).

(10 marks)

(c) The probability that a computer running a certain operating system crashes on any given day is 0.05.

(i) Find the probability that the computer crashes for the first time on the 10^{th} day after the operating system is installed.

(3 marks)

(ii) Find the mean and variance of days the computer runs before it crashes for the first time on any given day.

(3 marks)

(d) In the inspection of a fabric produced in continuous rolls, the number of imperfections per meter is a random variable X with $\lambda = 0.25$.

(i) What is the appropriate probability distribution for X?

(2 marks)

(ii) Find the probability that there are 2 imperfections found in 3 meters.

(3 marks)

Q2 (a) State two properties of normal curve.

(2 marks)

(b) Given that the moment generating function of an exponential random variable X with parameter θ is

$$M(t) = \frac{1}{1-\theta t} \text{ for } t < \frac{1}{\theta}.$$

Find the mean and variance of an exponential random variable.

(8 marks)

(c) A lawyer commutes daily from his suburban home to his midtown office. The average time for a one-way trip is 24 minutes, with a standard deviation of 3.8 minutes. Assume that the distribution of trip to be normally distributed.

(i) If the office opens at 9.00 am and he leaves his house at 8.45 am daily, what is the probability that he is late for work?

(3 marks)

(ii) Find the length of time above which we find the slowest 33% of the trips.

(3 marks)

(d) In a certain city, daily consumption of electric power, in millions of kilowatt-hours, is a random variable X having a gamma distribution with mean $\mu = 6$ and variance $\sigma^2 = 12$.

(i) Find the values of α and β.

(4 marks)

(ii) If the power plant of this city has a daily capacity of 12 million kilowatt-hours, what is the probability that this power supply will be inadequate on any given day?

(5 marks)

Q3 (a) Define a random sample.

(2 marks)

(b) Given a random sample of size n from a population that has the known mean μ and the finite variance σ^2, show that $E(S^2)$ is an unbiased estimator of σ^2.

(5 marks)

(c) If $X_1, X_2, ..., X_n$ constitute a random sample of size n from a normal population with the mean μ and the variance σ^2, derive the maximum likelihood estimates of μ.

(9 marks)

(d) The size of tea farms located at Kundasang, Sabah and Cameron Highlands, Pahang are presented as in Table **Q3(d)**.

Table Q3(d): The size of tea farms

Location	1	2	3	4	5	6	7	8
Kundasang	224	270	400	444	590	660	700	680
Cameron Highlands	116	96	239	329	427	597	689	576

Find the 90% confidence interval for the difference between two means. Assume that the two populations are normally distributed with unequal variances.

(9 marks)

Q4 (a) Use the Neyman-Pearson lemma to indicate how to construct the most powerful critical region of size α to test the null hypothesis $\theta = \theta_0$, where θ is the parameter of a binomial distribution with a given value of *n*, against the alternative hypothesis $\theta = \theta_1 < \theta_0$.

(6 marks)

(b) Suppose that we want to test the null hypothesis that an antipollution device for cars is effective.

(i) Explain under what conditions we would commit a type I error and under what conditions we would commit a type II error.

(2 marks)

(ii) Whether an error is a type I error or a type II error depends on how we formulate the null hypothesis. Rephrase the null hypothesis so that the type I error becomes a type II error.

(1 marks)

(c) The average weekly losses of work-hours due to accidents in 10 industrial plants before and after a certain safety program was put into operation were recorded as presented in Table **Q4(c)**.

Table Q4(c): Average weekly losses of work-hours

Before	45	73	46	124	33	57	83	34	26	17
After	36	60	44	119	35	51	77	29	24	11

(i) Test whether the safety program is effective at $\alpha = 0.05$.

(8 marks)

(ii) Test at $\alpha = 0.10$ whether it is reasonable to assume that the population has equal variances.

(8 marks)

- END OF QUESTION -