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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2013/2014

COURSE NAME	:	ENGINEERING MATHEMATICS IIE
COURSE CODE	:	BWM10303/BEE11403
PROGRAMME	:	1/2 BEV, 1/2/4 BEJ, 4BEE
EXAMINATION DATE	:	DECEMBER 2013/JANUARY 2014
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **FIVE (5)** QUESTIONS ONLY

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

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Q1 (a) Given the following periodic function,

$$f(t)=\begin{cases}-2, & -\pi \le t \le 0\\ 2, & 0 \le t \le \pi\end{cases}$$

$$f(x)=f(x+2\pi)$$

(i) Sketch the periodic function above for the interval $[-4\pi, 4\pi]$. Determine whether the above periodic function is an odd function, even function or neither odd nor even function.

(ii) Determine the Fourier series expansion to represent the above periodic function .

(14 marks)

(b) By referring to the Fourier transform pair table, evaluate

(i) $\mathcal{F}\{\ 4\delta(t+\pi)\}$,

(ii) $\mathcal{F}\{\ e^{4t}\sin(\pi t)H(t)\}$

(iii) $\mathcal{F}\{\ t^4 e^{-2t}H(t)\}$

(6 marks)

Q2 Given the initial value problem $y''-xy'+4y=0$, $y(0)=1$, $y'(0)=0$.

(a) By assuming $y=\sum_{m=0}^{\infty} c_m x^m$, show that the differential equation $y''-xy'+4y=0$ can be expressed as

$$\sum_{m=2}^{\infty} m(m-1)c_m x^{m-2} - \sum_{m=0}^{\infty} m\, c_m x^m + 4\sum_{m=0}^{\infty} c_m x^m = 0$$

(4 marks)

(b) Hence, by shifting the indices, show that reccurance relation is given by

$$c_{n+2}=\frac{(n-4)c_n}{(n+2)(n+1)}, \quad n=0, 1, 2, 3, \ldots$$

(7 marks)

(c) Then, deduce the coefficient of series for c_n, $n=0, 1, 2, 3, 4, 5, 6$ in term of c_0 and c_1. Hence, find that the general solution of the differential equation $y''-xy'+4y=0$

(7 marks)

(d) Given the initial condition $y(0)=1$ and $y'(0)=0$, find the particular solution of the differential equation $y''-xy'+4y=0$.

(2 marks)

Q3 (a)

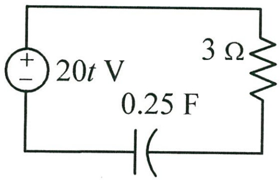

FIGURE Q3(a)

Figure **Q3(a)** shows a *RC* circuit. By applying Kirchhoff's Law, the *RC* circuit can be modelled as $3\frac{di}{dt}+4i=20$.

Given $i(0)=0$. Solve the governing equation by **TWO** different methods.

(12 marks)

(b) Solve $\frac{x^4+y^4}{x^5}dx-\frac{y^3}{x^4}dy=0$

(8 marks)

Q4 (a) Find the general solution for $\frac{d^2y}{dx^2}+4\frac{dy}{dx}=x^2+2$.

(10 marks)

(a) Given the homogeneous linear system

$$\boldsymbol{Y}'=\begin{pmatrix}10 & -5\\ 8 & -12\end{pmatrix}\boldsymbol{Y}.$$

Find the general solution of the corresponding homogeneous system above.

(10 marks)

Q5 (a) Evaluate $\mathcal{L}\{\, t^2 \sinh 4t\}$

(5 marks)

(b) Evaluate $\mathcal{L}\{2t-(2t+1)u(t-3)\}$

(8 marks)

(a) Evaluate $\mathcal{L}^{-1}\left\{\dfrac{s}{(s+4)^2(s-2)}\right\}$

(7 marks)

Q6 (a) Evaluate $\mathcal{L}^{-1}\left\{\dfrac{1}{s^2(s-1)^2}\right\}$ by using convolution theorem.

(10 marks)

(b) Solve the following differential equation by transforming into Laplace transform.

$$\frac{d^2y}{dt^2}+y=3\delta(t-2\pi),\ y(0)=0,\ y'(0)=0$$

(10 marks)

- END OF QUESTION -

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FORMULAS

Second-order Differential Equation

Characteristic equation: $am^2 + bm + c = 0$.		
Case	The roots of characteristic equation	General solution
1.	Real and different roots: m_1 and m_2	$y = Ae^{m_1 x} + Be^{m_2 x}$
2.	Real and equal roots: $m = m_1 = m_2$	$y = (A + Bx)e^{mx}$
3.	Complex roots: $m_1 = \alpha + \beta i$, $m_2 = \alpha - \beta i$	$y = e^{\alpha x}(A\cos \beta x + B\sin \beta x)$

Laplace Transform

$f(t)$	$F(s)$
a	$\frac{a}{s}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2 + a^2}$
$\cos at$	$\frac{s}{s^2 + a^2}$
$\sinh at$	$\frac{a}{s^2 - a^2}$
$\cosh at$	$\frac{s}{s^2 - a^2}$
t^n, $n = 1, 2, 3, \ldots$	$\frac{n!}{s^{n+1}}$
$e^{at} f(t)$	$F(s-a)$
$t^n f(t)$, $n = 1, 2, 3, \ldots$	$(-1)^n \frac{d^n}{ds^n} F(s)$
$H(t-a)$	$\frac{e^{-as}}{s}$
$f(t-a)H(t-a)$	$e^{-as} F(s)$
$\delta(t-a)$	e^{-as}
$\int_0^t f(u)g(t-u)\,du$	$F(s) \cdot G(s)$
y	$Y(s)$
y'	$sY(s) - y(0)$
y''	$s^2 Y(s) - sy(0) - y'(0)$

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Fourier Series

Fourier series expansion of periodic function with period 2π	Half Range series
$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$	$a_0 = \frac{2}{L}\int_0^L f(x)\,dx$
$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$	$a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}\,dx$
$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$	$b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}\,dx$
$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n\cos nx + \sum_{n=1}^{\infty} b_n\sin nx$	$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n\cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n\sin\frac{n\pi x}{L}$

Table of Fourier Transform (Fourier Transform Pairs)

$f(t)$	$F(\omega)$	$f(t)$	$F(\omega)$
$\delta(t)$	1	$\operatorname{sgn}(t)$	$\frac{2}{i\omega}$
$\delta(t-\omega_0)$	$e^{-i\omega_0\omega}$	$H(t)$	$\pi\delta(\omega)+\frac{1}{i\omega}$
1	$2\pi\delta(\omega)$	$e^{-\omega_0 t}H(t)$ for $\omega_0>0$	$\frac{1}{\omega_0+i\omega}$
$e^{i\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	$t^n e^{-\omega_0 t}H(t)$ for $\omega_0>0$	$\frac{n!}{(\omega_0+i\omega)^{n+1}}$
$\sin(\omega_0 t)$	$i\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$	$e^{-at}\sin(\omega_0 t)H(t)$ for $a>0$	$\frac{\omega_0}{(a+i\omega)^2+\omega_0^2}$
$\cos(\omega_0 t)$	$\pi[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)]$	$e^{-at}\cos(\omega_0 t)H(t)$ for $a>0$	$\frac{a+i\omega}{(a+i\omega)^2+\omega_0^2}$
$\sin(\omega_0 t)H(t)$	$\frac{\pi}{2}i[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]+\frac{\omega_0}{\omega_0^2-\omega^2}$		
$\cos(\omega_0 t)H(t)$	$\frac{\pi}{2}[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)]+\frac{i\omega}{\omega_0^2-\omega^2}$		