Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2014/2015

COURSE NAME	:	ENGINEERING MATHEMATICS IV
COURSE CODE	:	BDA 34003/ BWM 30603
PROGRAMME	:	3 BDD
EXAMINATION DATE	:	JUNE 2015/JULY 2015
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWER **ALL** QUESTIONS IN **SECTION A** 2. ANSWER **ONE** QUESTION IN **SECTION B** 3. PERFORM ALL CALCULATION IN **4 DECIMAL PLACES**

THIS QUESTION PAPER CONSISTS OF **TEN(10)** PAGES

SECTION A

Q1 The heat transfer performance of a new conductor material is under inspection. The material is fully insulated such that the heat transfer is only one dimensional in axial direction (x-axis). The initial temperature of the material is at room temperature of 25°C. At one end (point A) is heated while another end (point E) is attached to a cooler system, as shown in **Figure Q1**.

The unsteady state heating equation follows a heat equation

$$\frac{\partial T}{\partial t} - K\frac{\partial^2 T}{\partial x^2} = 0,$$

where K is thermal diffusivity of the material and x is the longitudinal coordinate of the bar. The thermal diffusivity of the steel is given as $K = 8$ mm^2/s, $\Delta x = h = 10$ and $\Delta t = k = 4$.

(a) Propose the heat equation for the new material in explicit finite-difference form in order to determine the temperature of point A, B, C, D and E, for every 4 seconds.

(4 marks)

(b) Sketch the general molecule graph.

(2 marks)

(c) Draw finite difference grid to predict the temperature of point A, B, C, D and E up to 8 seconds. Label all unknown temperatures in the grid.

(4 marks)

(d) Determine the unknown temperatures of point A, B, C, D and E at 8 seconds.

(10 marks)

Q2 The longitudinal vibration of a 2 meter bar which is fixed at both ends is governed by

$$\frac{\partial^2 u}{\partial t^2} = 9\frac{\partial^2 u}{\partial x^2}, \quad 0 \le t \le 0.04$$

where $u(x,t)$ is the axial displacement. The axial displacement of the bar has 5 evenly distributed assessment points, as shown in **Figure Q2**. The bar is then released with initial velocity which vibrates the bar. The initial displacements at the assessment points are tabulated in **Table 1**.

Table 1: Initial displacement at the assessment points

	Point A	Point B	Point C	Point D	Point E
x-coordinate, m	0	0.5	1	1.5	2
Displacement, m	0	0.1875	0.25	0.1875	0

If the axial displacement at $t = 0.02$ is governed by:

$$u_{i,j+1} = 0.0072u_{i-1,j} + 0.9856u_{i,j} + 0.0072u_{i+1,j} + 0.02\sin(2\pi x_i)$$

and the axial displacement at $t = 0.04$ is governed by:

$$u_{i,j+1} = 0.0144u_{i-1,j} + 1.9712u_{i,j} + 0.0144u_{i+1,j} - u_{i,j-1}$$

(a) Sketch the molecule graphs for $t = 0.02$ and $t = 0.04$.

(7 marks)

(b) Draw the analysis grid in order to analyze the displacements of all assessment points (A, B, C, D, E) for $t = 0\,\text{s}$, $t = 0.02\,\text{s}$ and $t = 0.04\,\text{s}$, if given that $u(0,t) = u(2,t) = 0$. Label all unknown displacements in the grid.

(5 marks)

(c) Determine the displacements of all points at $t = 0.02$ s and $t = 0.04$ s.

(8 marks)

Q3 (a) The upward velocity of a rocket is represented as a function of time in **Table 2**.

Table 2: The upward velocity of rocket at different time

Time, t	Velocity, v (m/s)
1	90.5
5	102.5
10	190

The velocity of the rocket can be approximated by the quadratic polynomial, where

$$v(t) = at^2 + bt + c$$

(i) Construct the corresponding system of linear equations.

(3 marks)

(ii) Solve the system of linear equations with Doolittle method.

(11 marks)

(iii) Justify whether the obtained system of linear equations in part (i) guarantee the convergence of iterative processes with the Gauss Seidel method.

(2 marks)

(b) The pressure p, acting on a piston of diameter 4cm in an internal combustion engine at different positions x is given in **Table 3**.

Table 3: Pressure acting on a piston at different position

Position x, cm	0	2	4	6	8
Pressure $p(x)$, N/cm^2	110	140	185	215	135

Compute the pressure difference rate at position 4 cm by using

(i) 2-point forward difference

(ii) 3-point central difference

(4 marks)

Q4 (a) Real mechanical systems may involve the deflection of nonlinear springs. As shown in **Figure Q4(a)**, a mass m is released from a distance h above a nonlinear spring. Conservation of energy can be used to show that

$$\frac{2k_2 d^{5/2}}{5} + \frac{1}{2}k_1 d^2 - mgd - mgh = 0$$

Given the parameter values $k_1 = 50,000g/s^2$, $k_2 = 40g/(s^2 m^5)$, $m = 90g$, $g = 9.81m/s^2$, and $h = 0.45m$.

Identify the value of d that satisfies the conservation of energy by using Newton-Raphson method with initial guess of $d_0 = 0.15$. Iterate the computation until $|d_{i+1} - d_i| \leq 0.0001$.

(10 marks)

(b) **Table 4** is generated by the function $f(x) = \dfrac{1}{1+x^2}$.

Table 4: Data generated by $f(x) = \dfrac{1}{1+x^2}$

x	0	1	2	3	7	9	12
$f(x)$	1	0.5	0.2	0.1	0.02	0.0122	0.0069

(i) Find the value of $f(x)$ when $x = 8.81$ analytically.

(2 mark)

(ii) Approximate the value of $f(x)$ when $x = 8.81$ by using linear Lagrange polynomial and cubic Lagrange polynomial.

(6 marks)

(iii) Analyze the suitability of using a cubic Lagrange polynomial to interpolate the value of $f(x)$ at $x = 12.23$.

(2 marks)

SECTION B

Q5 (a) A 11 meter beam is subjected to a load, and the shear force follows the equation

$$V(x) = 4 + 0.15x^2$$

where V is the shear force and x is length in meter along the beam. It is known that $V = \frac{dM}{dx}$, and M is the bending moment. Integration yields the relationship of

$$M = M_0 + \int_0^x V dx .$$

If M_0 is zero ,

(i) Evaluate M for $0 \leq x \leq 3$ by using Trapezoidal Rule, with suitable number of divisions.

(4 marks)

(ii) Evaluate M for $0 \leq x \leq 3$ by using Simpson's $\frac{1}{3}$ Rule, with suitable number of divisions.

(4 marks)

(iii) Considering the obtained answer in part (i) and (ii), justify which method gives higher accuracy.

(4 marks)

(b) Given

$$A = \begin{pmatrix} 4 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 4 \end{pmatrix}.$$

By taking $v^{(0)} = (1 \quad 1 \quad 0)^T$, find the smallest eigenvalue and its corresponding eigenvector by using shifted power method, if $\lambda_{largest} = 6.3165$. Iterate until $| m_{k+1} - m_k | < 0.005$.

(8 marks)

Q6 (a) Mechanical engineers are frequently faced with problems concerning the periodic motion of free bodies. Consider the simple pendulum problem as shown in **Figure Q6(a)**, its position at any time t can be described by:

$$\frac{d^2\theta}{dt^2} - \frac{g}{l}\theta = 0$$

where θ is the angular displacement, g is the gravitational constant, and l is the length of the weightless rod.

Given that $g = 9.81 m/s^2$, $l = 2m$ and the boundary conditions $\theta(0) = 0$ and $\theta'(0.3) = 2$, analyze the angle θ (in radian) for $0 \le t \le 0.3$ with $h = 0.1$ by using finite-difference method.

(12 marks)

(b) The rate of cooling of a body can be expressed as:

$$\frac{dT}{dt} = -k(T - T_a)$$

where T = temperature of the body in °C, T_a = temperature of the surrounding medium in °C and k = the proportionality constant (min^{-1}).

If a metal ball heated at 90°C is dropped into water that is held at a constant value of T_a = 20°C, determine its temperature at $t = 2$ min if $k = 0.2\ min^{-1}$ by using 4th order Runge-Kutta, with step length of $\Delta t = h = 1\, min$.

(8 marks)

- END OF QUESTION -

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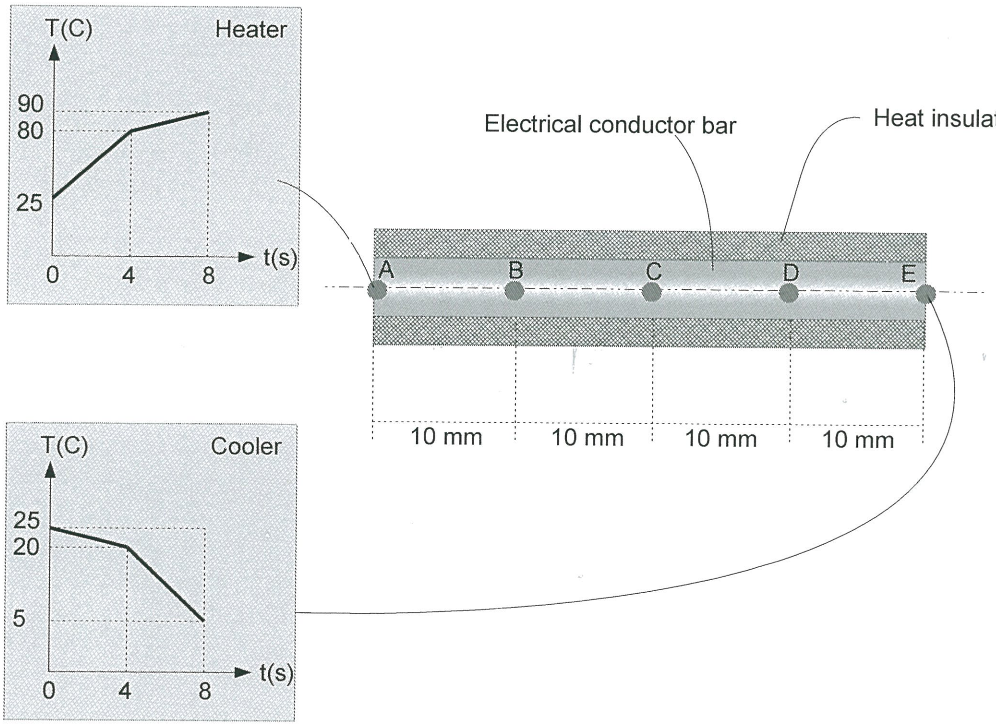

FIGURE Q1

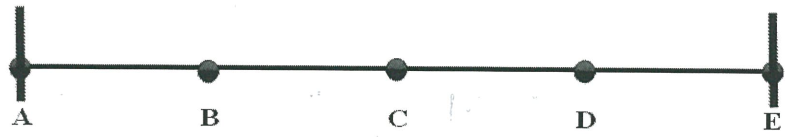

FIGURE Q2

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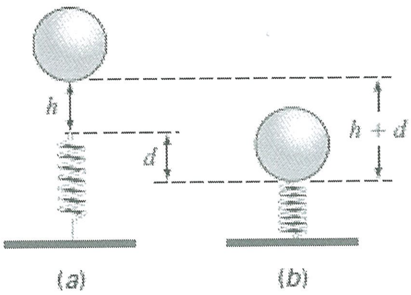

FIGURE Q4(a)

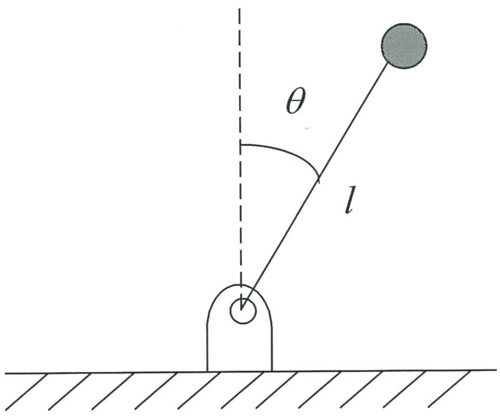

FIGURE Q6(a)

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FORMULAS

Partial differential equation

Heat Equation: Explicit finite difference method

$$\left(\frac{\partial T}{\partial t}\right)_{i,j} = \left(c^2 \frac{\partial^2 T}{\partial x^2}\right)_{i,j} \quad \Leftrightarrow \quad \frac{T_{i,j+1} - T_{i,j}}{k} = c^2 \frac{T_{i-1,j} - 2T_{i,j} + T_{i+1,j}}{h^2}$$

Nonlinear Equation

Newton Raphson Method: $x_{i+1} = x_i - \dfrac{f(x_i)}{f'(x_i)}$

Interpolation

Lagrange polynomial:

$$P_n(x) = \sum_{i=0}^{n} L_i(x) f(x_i), i = 0,1,2,\ldots,n \text{ where } L_i(x) = \prod_{\substack{j=0 \\ j\neq i}}^{n} \frac{(x - x_j)}{(x_i - x_j)}$$

Ordinary Partial Differential Equation

Runge Kutta Method

$$y_{i+1} = y_i + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

where

$$k_1 = hf(x_i, y_i)$$

$$k_2 = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_2}{2}\right)$$

$$k_4 = hf\left(x_i + h, y_i + k_3\right)$$

Boundary value problems:

Finite-difference method: $y_i' \approx \dfrac{y_{i+1} - y_{i-1}}{2h}$, $\quad y_i'' \approx \dfrac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$